MATHEMATICAL MODELLING AND ANALYSIS OF SIX-PHASE (MULTI-PHASE) POWER SYSTEMS

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By
SHRI NIWAS TIWARI

to the
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INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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dedicated to

my parents

EE-1982-D-TIW-MAT

CERTIFICATE

It is certified that this work, 'MATHEMATICAL MODELLING AND ANALYSIS OF SIX-PHASE (MULTI-PHASE) POWER SYSTEMS' by Shri Niwas Tiwari has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

Dr. L.P. Singh
Professor
Department of Electrical Engineering
Indian Institute of Technology
Kanpur, India.

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S.N. Tiwari

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Z_p^n, Y_p^n	phase impedance and admittance matrix of n-phase element respectively
Z_s^n, Y_s^n	sequence component impedance and admittance of n-phase element respectively
V_p^n, I_p^n	voltage and current vector of order n respectively
T_s^n, T_c^n	symmetrical and Clarke's component transformation matrix for n-phase system respectively
$Z^n_{p,eq}, Y^n_{p,eq}$	n-phase equivalent impedance and admittance matrix respectively of order (nxn)
A^n, B^n, C^n, D^n	ABCD-parameters of n-phase system
$[z], [y]$	leakage impedance and admittance matrix of a multi-phase transformer of appropriate dimensions respectively
y_{PS}, y_{PT}, y_{ST}	short circuit p.u. admittances (of a three winding transformer) of the two windings indicated by the subscripts with the third winding open respectively
P, S, T	primary, secondary and tertiary equivalent three-phase windings of a 3-φ/6-φ transformer respectively
α_T, β_T	turns ratio of the primary ($1+t_P$) and secondary ($1+t_S$) winding respectively
t_P, t_S	off-nominal tappings of primary and secondary winding respectively
z_1, z_2	leakage impedance of transformer 1 and 2 respectively
Z_p^1, Y_{sh}^1	per-phase series impedance and shunt admittance of a six-phase line (in p.u.) respectively
U^n	unity matrix of order (nxn)
α	$\exp(j2\pi/6)$
η	transmission line efficiency

H	inertia constant
P_{ei}	electrical power delivered by ith generator
P_{mi}	mechanical input power of machine i
ω	$2\pi f$
f	frequency (Hz)
δ	mechine rotor angle
t	time in sec.

Subscripts

S,R	sending and receiving end of a line
s	symmetrical component
TR	transformer
TL	transmission line
L	load
G	generator
p	phase
o	prefault/zero sequence
F	fault

Superscripts

T	transpose of a matrix or vector
n	phase or sequence order
*	conjugate of a complex number

Prefix

$\frac{d}{dt}$	differential operator

In additions, the symbols are explained in the text as and when it is necessary.

SYNOPSIS

S.N. TIWARI
Ph.D.
Department of Electrical Engineering
Indian Institute of Technology, Kanpur
April, 1982
MATHEMATICAL MODELLING AND ANALYSIS OF SIX-PHASE (MULTI-PHASE) POWER SYSTEMS

Because of everincreasing power supply requirements, electric utilities are facing the pressing need to transfer more and more power from generating stations to load centers. The most common solution of increasing power transmission capability by increasing transmission system voltages seems to have reached the point of saturation. High-voltage transmission lines present strong electric field at the ground surface with possible biological effects, visual pollution, audible noise and increasingly difficult problems in acquiring new rights-of-way etc. As a result, several alternatives for the purpose are being considered. Recently, the concept of multi-phase (high phase order) transmission was pioneered by Barnes and Barthold in place of conventional three-phase systems. Multi-phase transmission utilizes difficult to obtain rights-of-way in a better manner and offers a very appealing and unique solution to the problem of increased demand for electric energy. Since the initiation of the concept in 1972, the investigation on the use of multi-phase transmission has received growing interest.

In addition to multi-phase transmission, the use of multi-phase windings in generator construction has also been considered for the last several years. A six-phase generator (with associated six-phase/three-phase transformer) has been proposed for high power applications in several recent publications. Such generators have been found to be effective in reducing the effects of high armature circuit currents with the added advantage of reduced generated phase voltages and thus reduced insulation requirements in armature slots. The potential benefits of multi-phase system accrue from the smaller phase angles between the phases resulting in lower adjacent phase-phase voltages. Thus in a multi-phase transmission, the interphase insulation requirements, spacing, conductor surface gradient and noise levels are reduced considerably. As a result, a multi-phase line with smaller dimension can be used to transmit a larger amount of power covering the entire range of transmission voltages.

The general feasibility of multi-phase systems has been investigated in U.S.A. and the efforts have been made to unify the definitions of system voltages. At present, the six-phase system appears to be very promising among the multi-phase systems. In addition to investigation involving the steady state and transient operation of six-phase systems, overvoltages and insulation requirements, feasibility studies for converting certain existing double circuit three-phase

lines to six-phases have been carried out. The construction of experimental lines and their testing for six-phase and twelve phases are reported to be undertaken by certain research establishments such as P.T.I., NY (USA).

Fault analyses form an important step in the design of adequate protective schemes. A detailed fault analysis program employs symmetrical and Clarke's component transformations. Such transformations for multi-phase system, particularly the six-phase system are available in the literature. Employing these transformations, fault analysis of six-phase system has been discussed in several papers considering either a six-phase line conceptually in isolation or by focussing attention on a specific line in the system with the rest of the network represented by Thevenin's equivalent on either end of the line. From practical viewpoints, it would be desirable to include such details viz. proper fault impedances/admittances, the interfacing transformer for six-phase conversion and adequate representation of the rest of the network so that the performance of the overall system during fault may be investigated. Suitable, techniques for handling more complex faults and their combinations need to be developed since the transformation methods become unwieldy for such cases involving unbalanced network.

Multi-phase transmission systems, particularly the six-phase lines have been modelled by their phase impedance/admittance matrices and ABCD-parameters. The associated transformers (three-phase/six-phase) have been represented without considering the leakage impedances of the windings. Employing this simplified transformer representation, equivalent three-phase representations of six-phase lines and loads were obtained for carrying out the steady-state analysis of a composite three-phase and six-phase system on three-phase basis. In similar fashion, the three-phase lines and loads were treated for the analysis of composite system entirely on six-phase basis. However, detailed modelling and realistic analytical tools are needed to effectively analyse and evaluate the performance and also the advantages of multi-phase systems which could meet many of the needs that have developed for the power systems in recent years.

The objectives of this thesis are :

1. To develop suitable mathematical models of various multi-phase elements with special emphasis on phase parameter descriptions so that a six-phase (multi-phase) and a composite three-phase and six-phase power system may be represented in adequate details under balanced as well as under unbalanced conditions.

2. Detailed investigations of load flow, short circuit and transient stability problems employing the mathematical models developed in this work. The methods employing positive sequence as well as phase coordinate representations have been used.

An outline of the work reported in this thesis is given below :

1. An overview of the feasibility of multi-phase systems mainly based on the findings reported in the literature is presented to bring out the relative merits, demerits of multi-phase system and its compatibility with the conventional three-phase system.

2. The mathematical models of various multi-phase elements, viz., six-phase generator, three-phase/six-phase and six-phase transformers, six-phase lines and loads are developed. The transformers required for conversion from three phase to six-phase are represented to include the leakage impedances and off nominal tappings. An alternative model of the transformers (such as wye/star, delta/star and star/star) suitable for unbalanced network analysis is developed making use of the symmetrical lattice equivalent circuits. Employing the improved transformer models, equivalent three phase descriptions of a six-phase line in terms of phase impedance/admittance matrix

and ABCD-parameters are also derived. Similarly, the six-phase equivalent of a three-phase line is developed. Representation of multi-phase loads and their three-phase equivalents and the multi-phase equivalents of three phase loads in terms of phase and sequence parameters are obtained. Employing the element models, a six-phase (or composite three-phase and six-phase) power system is then modelled : in balanced conditions on an equivalent single-phase basis and in unbalanced conditions as an equivalent three-phase system; and also as a composite three-phase and six-phase system retaining the physical identities of different elements.

3. The procedure for load flow analysis for balanced as well as unbalanced conditions are developed for a completely six-phase and also for a composite three-phase and six-phase systems. One of the aspects studied in detail is the impact of converting certain existing double circuit three-phase lines to six-phase lines. The studies carried out on sample system indicate that multi-phase system, in general, have potentiality to maintain better voltage magnitudes, phase angle, and efficiency even for higher phase loadings than that of conventional three phase systems.

4. The fault analysis technique using symmetrical component transformation and bus impedance description of the network for three phase systems is generalized to handle completely six-phase or a composite three-phase and six-phase system. In addition, to investigate the behaviour of the system under unbalances and simultaneous faults, the method of phase coordinates is developed. One of the salient features of the study is a detailed ground fault investigation of a six-phase transmission system connected to the three-phase network via wye/star, three phase/six-phase transformers at both ends.

5. Several alternative schemes for transient stability analysis of six-phase and a composite three-phase and six-phase system are developed depending upon the interest of investigation either in three phase or six-phase or both three-phase and six-phase part of the network. Based on several case studies, performance of six-phase system is compared with the conventional three-phase systems.

CHAPTER 1

INTRODUCTION

1.1 MOTIVATION

The growing size and complexities of modern power systems have necessitated the refinements of design of transmission systems and/or search for better alternatives in view of the pressing needs to transfer more and more power, especially, from remote and large power plants to far away bulk load centers. There has been a phenomenal growth in design capabilities, and as a result, the transmission systems for voltages as high as 1100 kV are being planned. However, the most common solution to increase transmission capability by increasing transmission voltages of conventional three-phase systems seems to have reached the point of saturation. High voltage transmission lines present: strong electric field at the ground surface with possible biological effects, visual pollution, audible noise, and increasingly difficult problems in acquiring new rights of way etc. Even at HV and EHV levels, the problem of congested rights of way exists, especially, in urban areas and some of the common concerns of electric utilities are to economise the use of land and to increase the acceptability of overhead lines through reduced visibility and improved appearance. While these problems are being

tackled through improved design methods including compaction, aesthetic design etc. aimed at substantial reduction of sizes of multicircuit HV lines, the search and improvements of all viable alternatives is in progress.

Out of the various transmission alternatives, multi-phase (with phase order more than three) transmission is a unique approach to the problem of increasing transmission capability of overhead lines and may be applied over the full range of transmission voltages. Recently, six-phase generators have also been proposed for high power applications. These machines, if realised for practical applications, may advantageously be employed to energise multi-phase transmission systems. Studies have indicated that there are potential benefits that could accrue from the use of multi-phase systems. A multi-phase transmission becomes more attractive as a right of way utilization, and also, as the acquisition problems continue to grow and transmission capacity and economy increase in importance. Since the initiation of the concept in 1972 by Barthold and Barnes, the multi-phase transmission has been an active area of research for transmission planners and the publications in this area are rapidly increasing.

Most of the early works on multi-phase systems have been to investigate their feasibility of applications and examining their compatibility with three-phase systems. The encouraging results obtained from the preliminary investigations, a

systematic study of multi-phase system is receiving due attention. However, detailed modelling and realistic analytical tools are needed to effectively analyse and evaluate the performance and also the advantages of multi-phase systems which could meet many of the needs that have developed for power systems in recent years. The present thesis addresses itself to the mathematical modelling and analysis of six-phase (multi-phase) systems.

Before delineating the work reported in the thesis, a brief literature review of the various studies carried out on multi-phase systems is in order.

1.2 LITERATURE REVIEW

The concept of multi-phase (high phase order) transmission was initiated in 1972 at a CIGRE Meeting with the calculation of power density of the open wire transmission systems by Barthold and Barnes [1]. It was shown that the ultimate power density capability of open wire transmission is about 12,000 MW/m^2, which is several thousand times greater than is achieved by usual three-phase lines. This led to plan some alternatives, which might make up for higher space utilization of the right of way corridor. As a consequence, one such option was to increase the number of phases permitting increasing power transfer without increasing space requirements. Initial calculations [1] extended from 3 to 36 phases with

conductors symmetrically arranged in a circular configuration. Several features, namely increase in power capability, conductor gradients, electrostatic coupling, power frequency insulation, space requirements etc. were examined with respect to three-phase systems. The concept was further pursued by Venkata et al [2] and several aspects of six-phase transmission including power transfer capability, load flow, stability and reliability were examined in a preliminary manner. The feasibility of 138 kV six-phase transmission by converting a double circuit three-phase line to six-phase was reported by Guyker et al [3] highlighting several performance characteristics of 138 kV six-phase system, three-phase/six-phase transformer schemes, line tapping problem, overvoltages and protection schemes, substation, maintenance and operational considerations, etc.

A systematic and elaborate study of feasibility of multiphase systems was carried out by Stewart and Wilson in their two companion papers [4,5]. The feasibility analysis from steady-state considerations addressed definition of system voltage nomenclature, steady state operation as affected by; transmission line impedances, line and generator current unbalances, and the electrical environmental aspects of; radio noise, audible noise and electric field at the ground level [4]. The study from considerations of overvoltages and insulator requirements [5] evaluated those areas which influence phase

clearances and insulation requirements. Fault overvoltages, overvoltages due to interphase coupling, switching surges, rate of rise of circuit breaker recovery voltage and the lightning performance were analysed. In addition, the effects of wind, ice, fault currents, and phase spacer performance on conductor spacings were considered. Quantitative results for these areas were compared with representative three phase conditions in both the studies [4,5].

Venkata et al [9] have presented the description of the program for six-phase transmission line design highlighting the development of analytical tools for planning and evaluating the performance of a six-phase line. Stewart and Grant [7] have described studies for new lines based on six-phase and twelve phase test systems confirming analytical predictions of electrical and mechanical behaviours. It has been demonstrated that simple and pleasing structures can be built with available standard hardware. Grant et al [8] have reported the advances made in construction, testing, transient network simulation of switching surges etc. of the multi-phase transmission system.

The theory of symmetrical and Clarke's component transformations are wellknown to be important tools for carrying out fault analysis on balanced power system networks. The symmetrical component transformations for six-phase systems were derived by Bhatt et al [14] as straight forward extension of the three-phase technique. Using the same approach, symmetrical component

transformation for twelve phase systems were described by Stewart and Wilson [4]. Singh et al [21,22] have derived power invariant symmetrical and Clarke's component transformations for six and higher phase order systems employing group theoretic techniques and exploiting the symmetries inherent in power system networks. Willems [12,13,16] developed these transformations, conceptualising a six-phase system as two coupled three-phase systems. The Clarke's component transformations [16] thus derived can also diagonalize impedance matrix of six-phase transmission line which are not fully transposed. The method in [16] can be extended to systems having phase orders as multiples of three. Peeran et al [17] have described yet another generalization of Clarke's component transformation to six-phase systems.

The usual short circuit analyses of six-phase transmission systems were presented by Bhatt et al [14], Nanda et al [18], Willems [16] and Peeran et al [17] using the symmetrical and/or Clarke's component transformations derived by them. A considerable amount of effort in these studies [14,16,17,18] was made to explain the procedure by drawing sequence networks for each case of fault for conceptual clarity. Onogi and Okumoto [19] presented the fault analysis of a six-phase system by the combined use of two-phase and three-phase symmetrical component methods. The paper has described an interesting method of suppression of fault current in six-phase power transmission

systems [19]. Venkata et al [20] have discussed the various types of fault and their significant combinations which are likely to occur on a six-phase line. The analytical expressions for all 23 significant faults were presented. The results of fault analysis by focussing the attention on a specific 138 kV line in the power system network were presented in a tabular form. In addition, the technique for evaluating source impedances at the two ends of the line and the use of phase coordinate method for some types of fault were discussed [20].

Multi-phase transmission line, particularly the six-phase line has been modelled by phase impedance matrix by Bhatt et al [14], Willems [13,15,16] and Singh et al [21]. The problem of transposition of six-phase lines have been discussed by Stewart and Wilson [4], and Willems [16]. Willems [15] has also modelled the six-phase line by the ABCD-parameters similar to three phase line. Considering the fact that six-phase lines, whenever realised, will always be integrated in an otherwise three phase network, the mathematical descriptions of various three-phase/six-phase transformers were attempted by Willems [15]. For simplicity, the transformers were considered as ideal transformers and their phase coordinate and symmetrical component representations were obtained [15]. Employing these transformer models, three-phase equivalent representation of a six-phase line; both in impedance and admittance matrix form, and ABCD-parameters were derived to carry out the analysis of

the composite three phase and six-phase system entirely on three-phase basis. In addition, for carrying out the analysis of the composite system entirely on six-phase basis, the equivalent six-phase impedance/admittance matrix of a three phase line, was obtained.

Load flow and stability studies of six-phase systems remain largely unattempted except a preliminary investigation by Venkata et al in their paper on concept and reliability aspects of six-phase (multi-phase) power transmission systems[2]. Reliability aspects of six-phase transmission employing stochastic methods have been investigated by Venkata et al for a specific example [2] on a relative basis by comparing 138 kV double-circuit lines to 138 kV six-phase lines.

Use of six-phase generator along with its associated six-phase/three phase transformer for high power applications in three phase network was described by Holley et al [10] and Hanna et al [11]. Willems [13] has indicated that the six-phase generator and its associated transformer can be treated for fault analysis employing the symmetrical component representations using the transformation derived in [13]. A three-phase equivalent representation is suggested by combining the generator voltage and impedances with the equivalent diagram of the transformer [13].

After this brief historical review, a summary of the work

reported in the thesis is highlighted in the next section.

1.3 OBJECTIVES AND SCOPE OF THE THESIS

The objectives of this thesis are set-forth as follows :

1. To present an overview of the feasibility of multi-phase systems in general and six-phase systems in particular based on the findings reported in literature to bring out the performance characteristics, suitability of application and compatibility of the new technology with conventional three-phase system.

2. To develop suitable mathematical models of various multi-phase elements and overall system with special emphasis on phase parameter descriptions so that a six-phase and also a composite three-phase and six-phase power system may be represented in adequate details under balanced as well as unbalanced conditions.

3. Detailed investigations of planning studies viz. load flow, short circuit and transient stability employing mathematical models developed in this thesis. The methods employing positive sequence as well as phase co-ordinate representations are used depending upon the nature of the problem and scope of the studies.

4. To study the performance of six-phase systems with the aid of several case studies.

1.4 SUMMARY OF WORK

A chapterwise summary of the work reported in this thesis is as follows :

Chapter 2 initially presents an overview of the feasibility of multi-phase systems from various considerations including steady state operation, overvoltages and insulation requirements etc. The important findings of various studies reported in the literature are consolidated to bring out the relative merits and demerits of multi-phase systems and their performance characteristics. Then, the mathematical modelling of multi-phase elements, namely, six-phase generator, transformers (six-phase, and three-phase/six-phase) of different types, transmission lines, loads etc. are developed. Three-phase equivalent suitable for unbalanced analysis and single phase equivalent of multi-phase element are systematically derived. The phase co-ordinate representation of multi-phase elements of a composite three-phase and six-phase system retaining the physical identities of three-phase and six-phase element is presented. Since a multi-phase transmission line is looked upon as a network between three-phase buses of sending and receiving ends, its equivalent ABCD-parameters, and impedance/admittance matrices are different for different types of transformer connections at the two ends. The equivalent three-phase models are obtained in an unified general format to circumvent this difficulty.

Similarly the multi-phase equivalent of three-phase line is also derived. These equivalent representations are needed to carry out the analysis of the overall system either entirely on three phase or on six-phase (multi-phase) basis.

Chapter 3 is devoted to the various alternative representations of overall networks and the load flow studies of multi-phase systems. Employing the models of multi-phase elements (Chapter 2) along with the usual three-phase elements, a multi-phase (or composite three-phase and six-phase) power system is represented : in balanced conditions on a single phase basis, and in unbalanced conditions on an equivalent three-phase basis and also on a composite three-phase and six-phase basis, retaining the physical identities of different elements. In addition, the procedure of assembling a multi-phase bus admittance matrix is developed and explained through a flow chart and numerical example.

Employing the single-phase representation of the network, the balanced single phase load flow is carried out as usual. The phase coordinate load flow method is developed by modifying and extending the three-phase methods to include the suitable representation of multi-phase elements for indepth investigation of the problem of unbalances. Three alternative schemes are discussed to effectively analyse a composite three-phase and six-phase system depending upon whether the interest of investigation lies in the three-phase, or six-phase or both three-phase

and six-phase part of the network. A sample system is worked out to demonstrate the procedure and examine the validity of the programs developed for the purpose. In addition, several case studies are presented to evaluate the performance of multi-phase system including the impact of converting certain double circuit three phase lines to six-phase lines in the existing three phase system.

In Chapter 4, the fault analysis of multi-phase system is presented. Generalized schemes using the transformation technique suitable for faults on balanced system and phase coordinate technique suitable for all types of series, shunt and simultaneous faults on unbalanced system are derived as an extension of three-phase methods. The theoretical framework and the algorithm of the procedures are discussed. Various alternative schemes employing phase coordinate method as applicable for load flow studies are employed here as well. The procedure and the programs developed for the purpose are validated by working out a sample system. In addition, a detailed fault analysis of six-phase transmission line (forming the part of a composite three-phase and six-phase system) is carried out.

Chapter 5 is devoted to the transient state stability studies of multi-phase power systems. Three phase techniques are modified to account for the adequate representation of multi-phase system for this purpose. Finally,

a sample network is worked out to bring out the salient features of these studies.

Finally, Chapter 6 concludes with a review of the work done and outlines for further scope of research.

1.5 CONCLUDING REMARKS

In this chapter, a brief account of work previously carried out on multi-phase systems and the motivation for present investigation are presented. The objectives and scope of the thesis are set forth to deal with mathematical modelling and analysis of six-phase (multi-phase) systems with special emphasis on phase parameter descriptions and analysis. In addition, a chapterwise outline of the work reported in this thesis is given.

CHAPTER 2

AN OVERVIEW OF FEASIBILITY AND MATHEMATICAL MODELLING OF SIX-PHASE (MULTI-PHASE) ELEMENTS

2.1 INTRODUCTION

Electric power systems have largely developed as three-phase systems, although high phase order machine construction and power transmission have been considered for last several years [1-11]. The disadvantage of adding more phases than three, seems to be the main cause confronting the development of multi-phase systems. However, with the growth of increasingly sophisticated design methods and increased importance of economic, environmental and several other factors, the multi-phase systems are being considered as one of the potential alternatives to conventional three phase systems. The benefits of multi phase system accrue from the smaller phase angle between the phases resulting in lower adjacent phase-phase voltages. Multi-phase winding arrangements have been found to be very effective in building large turbine generators [10]. A six-phase generator associated with six-phase/three-phase transformer has been proposed for high power applications in several publications [10,11]. Such generators are considered to be especially attractive for feeding power to a HVDC transmission grid because of their reduced sensitivity to harmonics in line current that might be introduced by rectifier systems[10].

With the use of higher phases in transmission, the inter-phase insulation requirements, spacings, conductor surface gradient and noise levels are reduced considerably [3-9]. As a result, a multi-phase line with smaller dimension can be used to transmit a larger amount of power covering the entire range of transmission voltages. However, the basic feasibility of this new technology under various operating conditions must be established before the actual planning of such systems. Although the research in this area is still in its infancy, yet some extremely important findings have been reported in the literature [2-9] indicating general feasibility of multi-phase systems. In order to obtain an overall picture, Section 2.2 is devoted to present an over-view of the feasibility of multi phase systems.

Mathematical modelling is a very flexible and in-expensive tool needed for indepth and detailed analysis of the behaviour of power systems both from the view points of planning and operation. In view of the growing interest in the multi phase system and examining its compatibility with existing three phase system, the development of suitable mathematical models of elements of multi phase systems will greatly benefit such studies. Earlier, multi-phase trans-mission lines, particularly, the six-phase lines have been described by their phase impedance/admittance matrices by Bhatt et al [14], Singh et al [21,22] and Willems [16] and

by ABCD-parameters by Willems [15,16]. Considering a six-phase line interfaced to the three-phase network via three phase/six-phase transformers, Willems [15] developed equivalent three-phase descriptions in terms of ABCD-parameters and impedance/admittance matrices. Similarly, the six-phase equivalent of a three-phase line were derived. The various transformer connection schemes for six-phase conversion were considered without the leakage impedance of windings for simplicity. A realistic analysis of multi-phase system requires adequate representation of various elements. With this in view, the transformer models are improved by including leakage impedances and also including the off nominal tappings. An alternative model of three-phase/six-phase transformers suitable for unbalanced network analysis is developed employing the symmetrical lattice equivalent circuits in Section 2.3.2. Employing the improved models of transformers, three-phase equivalent representation of six-phase line and six-phase equivalent of three-phase line are developed in Section 2.3.3. Multi-phase loads and their three-phase equivalents and multi-phase equivalents of three-phase loads are derived in Section 2.3.4. In addition, a six-phase generator model incorporating unbalances in the machine is derived in Section 2.3.1 which may be employed for unbalanced network analysis. Finally, the concluding remarks are presented in Section 2.4.

2.2 AN OVERVIEW OF FEASIBILITY OF MULTI-PHASE SYSTEMS

Basic feasibility studies of multi-phase transmission, and particularly the six-phase transmission have been reported for normal steady state operation, overvoltages and insulation requirements, powerflow and reliability etc. in several studies [2-5,7-8]. The studies reveal several interesting features of multiphase systems and, in several cases, a six-phase system has been found to possess better characteristics than those of its three phase counterparts. The important findings of these studies are consolidated here for ready reference and to bring out the relative merits and demerits of multi-phase systems.

(i) Power Transfer Capability [2]

The multi-phase systems offer increased power transfer capability with appreciable advantages in terms of smaller line and tower dimensions. The advantage in terms of uprating existing line is also substantial. For example, if an existing double circuit three-phase line is converted to a six-phase line maintaining the same adjacent phase-phase voltages in both the cases, the power transfer capability can be increased by 73.2 percent.

(ii) System Voltage Definition, and Phase-ground and Phase-Phase (adjacent) Voltages [4]

Out of the various possible definitions of system voltages, phase-ground voltage definition is favoured. This is in view of

the fact that the adjacent phase-phase voltage (for a circular configuration of conductors) decreases with phase order. For higher phase orders, this becomes less than the phase-ground voltages. However, for a six phase system both are equal. The Table 1 gives the relative picture of the phase-ground and phase-phase voltages for 3-24 phase system [4].

Table 2.1

Phase-ground and phase-phase voltages [4]

Phase-ground voltage (kV)	Phase-Phase (Adjacent Phases) Voltage			
	3-Phase (kV)	6-Phase (kV)	12-Phase (kV)	24-Phase (kV)
80	138	80	41	21
133	230	133	69	35
199	345	199	103	52
289	500	289	150	75
442	765	442	229	115

(iii) Conductor Clearance [5]

The line conductor spacings can be reduced with increasing phase order since the phase-phase voltage decreases for constant phase-ground voltage. The degree of reduction is limited by motion of the individual conductors owing to ice, wind and faults etc.

(iv) Surge Impedance Loading (SIL) [4]

The surge impedance loading is approximately proportional to the phase order increase and reaches saturation beyond six-phases. Therefore, if SIL is the circuit rating criterion, the phase order beyond six becomes questionable.

(v) Thermal Loading [4]

The thermal loading follows a straight line relationship with phase order. Thus if the thermal loading is the criterion for circuit rating, the capacity increase is proportional to the number of phases.

(vi) Positive Sequence Surge Impedance, Inductive Reactance, and X_0/X_1 Ratio [4]

These are somewhat higher for six-phase line than for three phase line.

(vii) Transposition [4,16]

While the three-phase lines can be freely transposed, the multi-phase lines are difficult to transpose. The only practical transposition for multi-phase lines is obtained by rotating the entire conductor array in steps over the length of the line.

(viii) Current Unbalance [4]

Multi-phase lines exhibit line current unbalance characteristics comparable to or better than their three-phase

counterparts. Six-phase circuits are better balanced as compared to two three-phase circuits with the same conductor configurations. In case of circular array, this is further improved and transposition may be unnecessary.

(ix) Electric fields [4]

The maximum surface electric field decreases with phase order, whereas, the maximum ground electric field increases with phase order. The addition of shield wires increases the surface electric field on the conductors and reduces the ground level field. The electric field for one conductor open and unfaulted (single pole switching) shows small variation as compared with the same conductor open and grounded.

(x) Radio and audible noise [4]

Radio and audible noise are reduced with phase order. The performance of a six-phase system is better than two three-phase circuits having the same number of conductors.

(xi) Fault overvoltages [5]

The fault overvoltages for six-phase systems are slightly higher than for a comparable three-phase system. For phase orders higher than six, the fault overvoltages are comparable to those of a three-phase system.

(xii) Switching Surges [5]

Phase-ground switching surges for three-phase and six-phase lines are approximately the same for the same conditions with less than 4 percent difference. The phase-phase surges become increasingly important relative to phase-ground surges as the phase order increases. This may also set up a practical upper limit to the phase order achievable.

(xiii) Rate of rise of recovery voltage (RRRV) [5]

RRRV across the breaker terminals during normal opening is found to be less on high phase order systems than for three-phase systems with comparable equivalent short circuit MVA.

(xiv) Lightning Performance [5]

It has been estimated that owing to the reduced dimension of six-phase line , the number of lightning strokes to the line will be of the order of 20 percent less than for the double circuit three phase line. For the same phase-ground voltage, conventional three-phase double circuit line and the compact six-phase circuit may have very similar back flash over performance. The probability of phase-phase flash over is much higher on a six-phase line. For well shielded lines, this will be of no significance. The overall lightning performance of high phase order lines will be comparable to conventional three-phase lines assuming effective shielding.

(xv) Terminal Insulation Level [5]

The terminal insulation level tend to be slightly higher for six-phase systems than for three-phase systems or for systems of 12 phases and higher.

(xvi) Reliability Aspect

A preliminary examination of this aspect from load flow, stability and stochastic reliability [2] for uprating double circuit three phase line to six-phase line suggests that :

(a) More demand at a load point can be met by converting a double circuit three phase line to a six-phase line for the same phase-phase (adjacent) voltage with a relatively less incremental cost and utilizing the same transmission corridor.

(b) Voltage regulation and efficiency are better with six-phase line.

(c) A six-phase line appears to be more stable than a three-phase double circuit line.

(d) More energy can be made available at a load point with six-phase line than with three-phase double circuit line.

(e) The loss of a six-phase line may have a greater impact on power system than a three-phase double circuit line.

Based on the potential benefits of multi-phase systems substantiated by the feasibility studies as discussed above, construction of experimental multi-phase lines and evaluating circuit uprating applications have been carried out by certain utilities [2,7-8]. These studies have been very successful and conclusive and may be summed up as follows :

(a) All the basic design information needed for uprating and new designs of six-phase lines are available.

(b) Test lines have successfully shown practical and simple construction for six or twelve phase overhead lines.

(c) Standard insulator and hardware components can be combined to provide simple and attractive conductor support systems that allow maximum advantage from compaction technology.

(d) A range of tower design has been developed and demonstrated on test lines. The small spacings possible with high phase order permit small, unobtrusive and attractive line designs.

(e) If multi-phase systems are developed with the same phase-ground voltages as are standard for three-phase, then multi-phase three-phase interconnections will be possible i.e. a high-phase order line can be completely compatible with a three phase system.

The above developments signify an important step towards the realization of multi-phase transmission system in the near future. The multi-phase transmission may find most economic applications in narrower rights of way, uprating and multi-circuit configurations in the entire transmission range of voltages.

2.3 MATHEMATICAL MODELLING OF MULTI-PHASE ELEMENTS

The mathematical models of various elements of multi-phase systems viz., machine, transformers, lines and loads are developed in this section for their later application to system studies including load flow and fault analysis.

2.3.1 Machine

Six-phase generators have been proposed for high power applications by Hanna et al [11] and Holley et al [10] and their integration to three phase network via six-phase/three-phase transformer schemes has been discussed. The feasibility of using high phase order machines is a topic that is still in its infancy. However, if these machines become a reality in future, they may be effectively employed to energise multi-phase transmission network. These machines can be modelled and analysed without much difficulties. For example, consider a six-phase synchronous generator (Fig. 2.1) grounded through a reactance Y_N. The detailed representation incorporating the imbalances in the machine is given by,

$$\begin{bmatrix} Y_p^6 & -\overline{Y}_1 & -\overline{Y}_o \\ -\overline{Y}_2 & 6y_1 & 0 \\ -\overline{Y}_o^T & 0 & (y_N+6y_o) \end{bmatrix} \begin{bmatrix} V_p^6 \\ E_a \\ V_N \end{bmatrix} = \begin{bmatrix} I_p^6 \\ S^*/E_a^* \\ I_N \end{bmatrix} \qquad (2.1)$$

The machine model vide eqn. (2.1) is derived in Appendix A and is based on the fact that the internal induced voltages of the generator are balanced whereas its terminal voltages which depend also on internal machine impedances and imbalance in machine currents may be unbalanced. Due to this imbalance, each phase power is not equal to one sixth of the total power thereby prohibiting the calculation of phase currents.

The phase admittance matrix Y_p^6 of the machine may be computed from the basic element data and geometry or by transformation of previously computed symmetrical component admittance/impedance quantities e.g.

$$Y_p^6 = [T_S^6]\,[Y_{comp}^6]\,[T_S^{*\,6}] \qquad (2.2)$$

where $Y_{comp}^6 = \text{Diag.}\,[y_o,\ y_1,\ y_2,\ y_3,\ y_4,\ y_5]$

and

$$T_S^6 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha^* & -\alpha & -1 & -\alpha^* & \alpha \\ 1 & -\alpha & -\alpha^* & 1 & -\alpha & -\alpha^* \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\alpha^* & -\alpha & 1 & -\alpha^* & -\alpha \\ 1 & \alpha & -\alpha^* & -1 & -\alpha & \alpha^* \end{bmatrix} \qquad (2.3)$$

is a symmetrical component transformation for six-phase systems [21].

It may be noted that the machine model eqn. (2.1) makes use of the total power and not the individual phase powers. The model (2.1) has been written neglecting the term $y_N |V_N|^2/E_a^*$ as discussed in Appendix A. The injected currents I_p^6 for generator buses are zero. However, if there are local loads on machine, they are not zero and one sixth of the total local load is assigned to each of the phases.

2.3.2 Transformers

The transformers associated with multi-phase systems are basically required to obtain 6,9,12 and higher order phase conversions from three phase systems. A six-phase conversion can be obtained from the commonly available three-phase units employing a variety of connection schemes [3,40] viz. wye/star, delta/star, wye/hexagon and star connected auto-transformers. The other worthwhile schemes may be zig-zag transformers, three winding transformers with one winding utilized for three-phase connection and other two for six-phase, and two three-phase units with appropriate phase reversal. The still higher phase conversion need specially constructed units. For the present purpose, only few three-phase/six-phase and a six-phase transformer are considered for developing their models and it is assumed that the other schemes may be treated in a similar manner.

Transformer Models I

The transformers are represented as ideal transformers in series with the equivalent p.u. leakage impedances/ admittances of the windings.

Three phase/six-phase Transformers

Consider a wye/star transformer shown in Fig. 2.2. A nominal ratio transformer may be represented by an ideal transformer in series with the equivalent p.u. leakage impedance z of the windings. The voltage and current relations at the terminals of the ideal transformer [15] are,

$$V'^{6}_{p} = N\, V^{3}_{p} \quad ; \qquad V^{3}_{p} = \tfrac{1}{2}\, N^{T}\, V'^{6}_{p} \tag{2.4}$$

$$I^{6}_{p} = N\, I^{3}_{p} \quad ; \qquad I^{3}_{p} = \tfrac{1}{2}\, N^{T}\, I^{6}_{p} \tag{2.5}$$

The terminal voltages of the transformer in terms of the induced voltage vector V'^{6}_{p} is given by

$$V^{6}_{p} = V'^{6}_{p} - [z]\, I^{6}_{p} \tag{2.6}$$

where

$$V^{3}_{p} = [V_a\ V_b\ V_c]^T$$

$$V^{6}_{p} = [V_1\ V_2\ V_3\ V_4\ V_5\ V_6]^T$$

$$I^{3}_{p} = [I_a\ I_b\ I_c]^T$$

$$I^{6}_{p} = [I_1\ I_2\ I_3\ I_4\ I_5\ I_6]^T$$

$$[z] = (6x6) \text{ order diagonal matrix } = [y]^{-1}$$

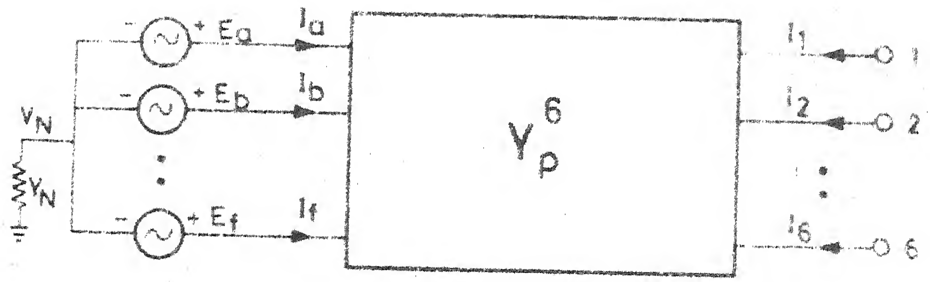


FIG. 2.1 SCHEMATIC REPRESENTATION OF A SIX-PHASE GENERATOR

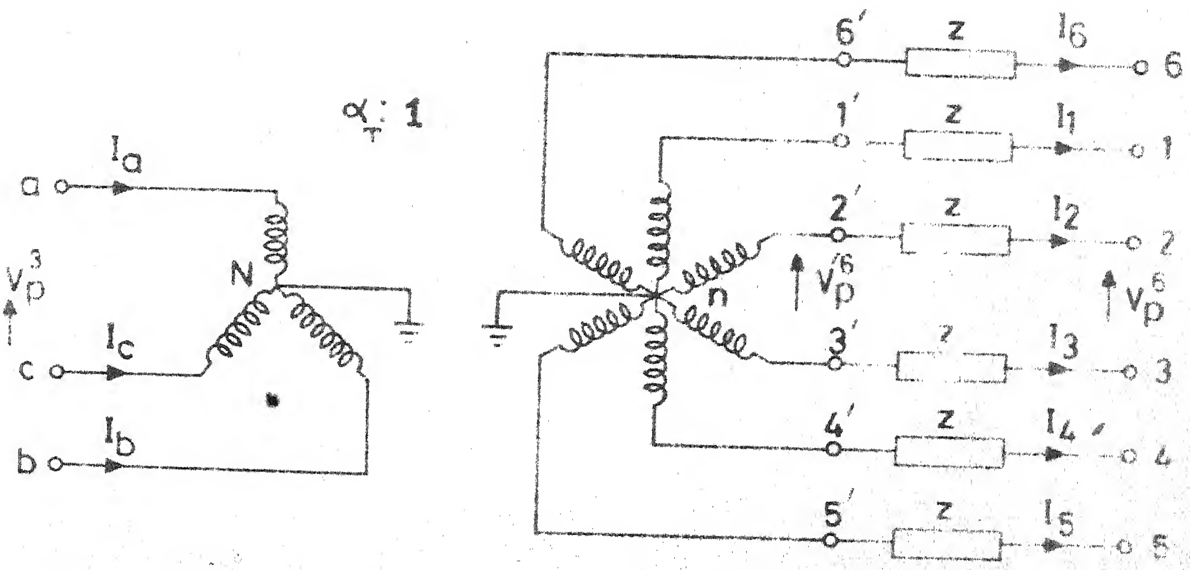


FIG. 2.2 A WYE/STAR, THREE-PHASE/SIX-PHASE TRANSFORMER OF TURNS RATIO α_T:1

and

$$N^T = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

From eqns. (2.6) and (2.4), I_p^6 is obtained as follows :

$$I_p^6 = [y]\ [N]\ V_p^3 - [y]\ V_p^6 \tag{2.7}$$

and from eqns. (2.5) and (2.7), I_p^3 is obtained as

$$I_p^3 = \tfrac{1}{2}\ [N]^T\ [y]\ [N]\ V_p^3 - \tfrac{1}{2}\ [N]^T\ [y]\ V_p^6 \tag{2.8}$$

The nodal representation of the transformer as obtained from eqns. (2.7) and (2.8) is given by

$$\begin{bmatrix} I_p^3 \\ \\ -I_p^6 \end{bmatrix} = \begin{bmatrix} \tfrac{1}{2}[N]^T\ [y][N] & -\tfrac{1}{2}\ [N]^T\ [y] \\ \\ -[y][N] & [y] \end{bmatrix} \begin{bmatrix} V_p^3 \\ \\ V_p^6 \end{bmatrix} \tag{2.9}$$

Equation (2.9) after expanding yields the nodal admittance matrix of order (9x9) of the transformer as :

$$
Y_{TR} = \begin{array}{c} \\ a \\ b \\ c \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\begin{array}{c}
\begin{array}{ccc|cccccc} a & b & c & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\left[\begin{array}{ccc|cccccc}
y & & & -y/2 & & & y/2 & & \\
 & y & & & & -y/2 & & & y/2 \\
 & & y & & y/2 & & & -y/2 & \\
\hline
-y & & & y & & & & & \\
 & & y & & y & & & & \\
 & -y & & & & y & & & \\
y & & & & & & y & & \\
 & & -y & & & & & y & \\
 & y & & & & & & & y
\end{array}\right]
\end{array}
\qquad (2.10)
$$

Equation (2.9) can be cast in a general format from which the nodal representations for different schemes may be obtained. In addition, the off nominal tapping could also be considered. Including the tappings on one of the winding (say three-phase side), and following the procedure as discussed above, such a representation is given by

$$
\begin{bmatrix} I_p^3 \\ -I_p^6 \end{bmatrix} = \begin{bmatrix} k_1[H]^T\,[y][H] & -k_2[H]^T\,[y] \\ -k_3[y][H] & [y] \end{bmatrix} \begin{bmatrix} V_p^3 \\ V_p^6 \end{bmatrix} \qquad (2.11)
$$

where the constants k_1, k_2 and k_3, and matrix H for different transformer connections are summarised in Table 2.2 and the terminal relationships of the transformers are given in Table 2.3.

Table 2.2

Constants k_1, k_2, k_3 and matrix H for different transformers

S.No.	Transformer type	Constants			Matrix H
		k_1	k_2	k_3	
1	Wye/star	$1/(2\alpha_T^2)$	$1/(2\alpha_T)$	$1/\alpha_T$	N
2	Delta/star	$1/(6\alpha_T^2)$	$1/(2\sqrt{3}\alpha_T)$	$1/(\sqrt{3}\alpha_T)$	NP
3	Wye/hexagon	$1/(2\alpha_T^2)$	$1/(2\alpha_T)$	$1/(\alpha_T)$	$Q^T N$
4	Delta/hexagon	$1/(6\alpha_T^2)$	$1/(2\sqrt{3}\alpha_T)$	$1/(\sqrt{3}\alpha_T)$	$Q^T NP$

Table 2.3

Terminal relations at the ideal transformer for different connection schemes

S.No.	Transformer type	Voltage relations	Current relations
1	Wye/star (Fig. 2.2)	$V_p^3 = \frac{\alpha_T}{2} N^T V_p'^6 \; ; \quad V_p'^6 = \frac{1}{\alpha_T} N V_p^3$	$I_p^3 = \frac{1}{2\alpha_T} N^T I_p^6 ; \quad I_p^6 = \alpha_T N \, I_p^3$
2	Delta/star(Fig.2.3)	$V_p^3 = \frac{\alpha_T}{2\sqrt{3}} P^T N^T V_p'^6 ; V_p'^6 = \frac{1}{\sqrt{3}\alpha_T} NPV_p^3$	$I_p^3 = \frac{1}{2\sqrt{3}\alpha_T} P^T N^T I_p^6 ; I_p^6 = \frac{\alpha_T}{\sqrt{3}} NPI_p^3$
3	Wye/hexagon(Fig.2.4)	$V_p^3 = \frac{\alpha_T}{2} N^T Q V_p'^6 ; \quad V_p'^6 = \frac{1}{\alpha_T} Q^T N V_p^3$	$I_p^3 = \frac{1}{2\alpha_T} N^T Q I_p^6 ; \quad I_p^6 = \alpha_T Q^T N I_p^3$
4	Delta/hexagon(Fig.2.5)	$V_p^3 = \frac{\alpha_T}{2\sqrt{3}} P^T N^T Q V_p'^6 ;$ $V_p'^6 = \frac{1}{\sqrt{3}\alpha_T} Q^T NPV_p^3$	$I_p^3 = \frac{1}{2\sqrt{3}\alpha_T} P^T N^T Q I_p^6 ; I_p^6 = \frac{\alpha_T}{\sqrt{3}} Q^T NPI_p^3$

where

$$P = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad Q = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Six-phase Transformer

A six-phase two winding transformer can be modelled similar to three-phase two winding transformer. For example, consider a star/star transformer depicted in Fig. 2.6.

The relationships at the ideal transformer terminals are given as,

$$\alpha_T \, V_O'^{6} = V_i^6 \quad , \quad \alpha_T \, I_i^6 = I_O^6 \tag{2.12}$$

$$V_O^6 = V_O'^{6} - [z] \, I_O^6 \tag{2.13}$$

From relations (2.12) and eqn. (2.13), the nodal admittance matrix description of the transformer is obtained as

$$\begin{bmatrix} I_i^6 \\ -I_O^6 \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha_T^2}[y] & -\frac{1}{\alpha}[y] \\ -\frac{1}{\alpha}[y] & [y] \end{bmatrix} \begin{bmatrix} V_i^6 \\ V_O^6 \end{bmatrix} \tag{2.14}$$

Transformer Model II (Equivalent Circuit Representation)

An alternative model suitable for unbalanced system analysis is developed employing symmetrical lattice equivalent

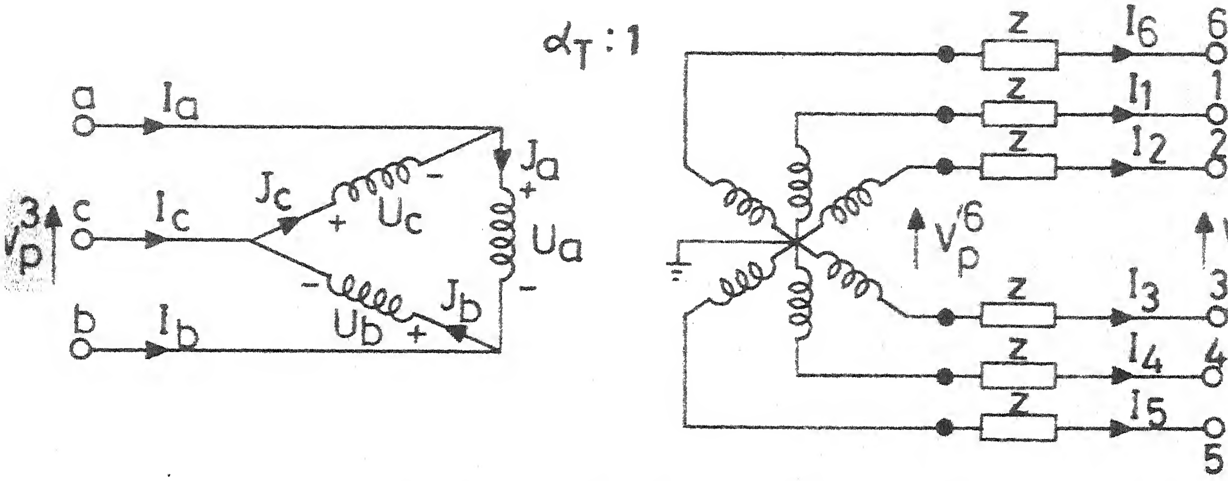


Fig. 2.3 A delta/star 3-ϕ/6-ϕ transformer of turns ratio $\alpha_T:1$

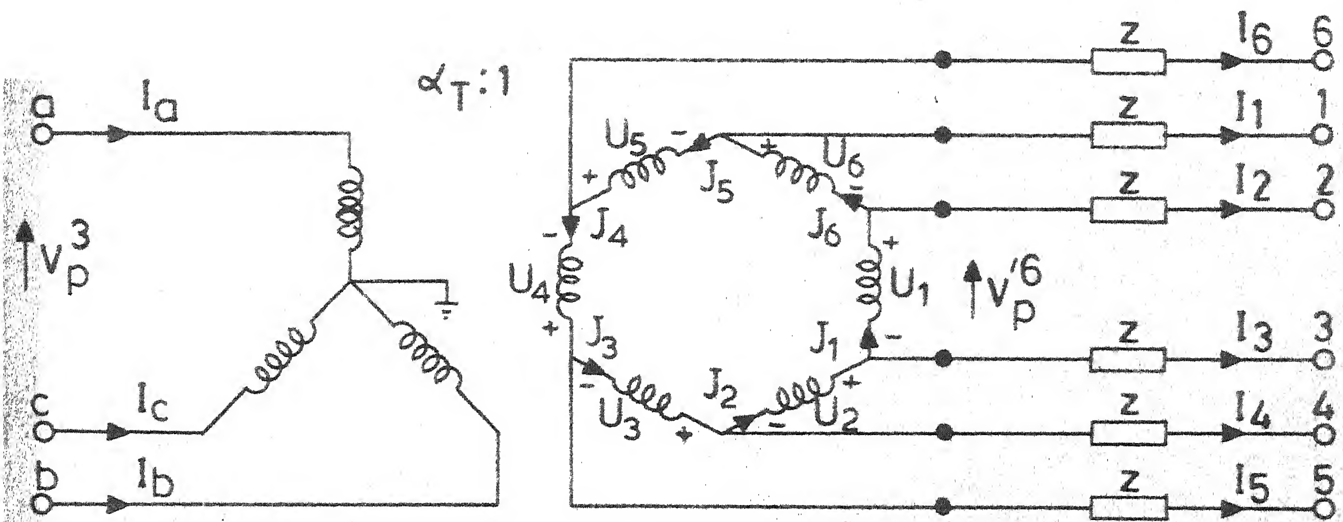


Fig. 2.4 A wye/hexagon 3-ϕ/6-ϕ transformer of turns ratio $\alpha_T:1$.

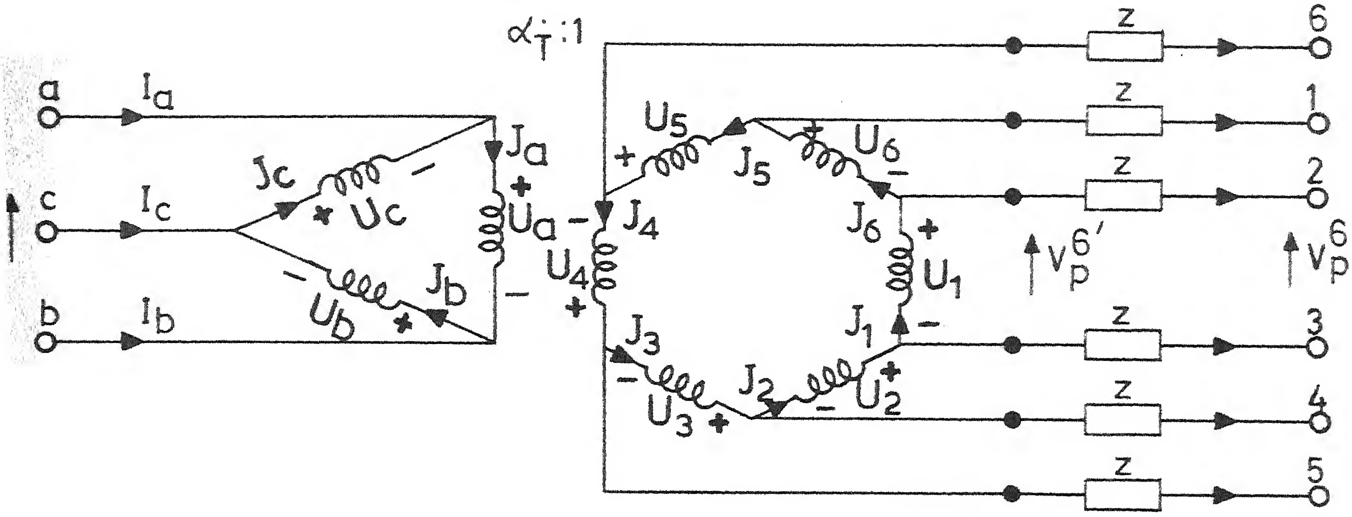


Fig. 2.5 A delta/hexagon 3-ϕ/6-ϕ transformer of turns ratio α_T:1.

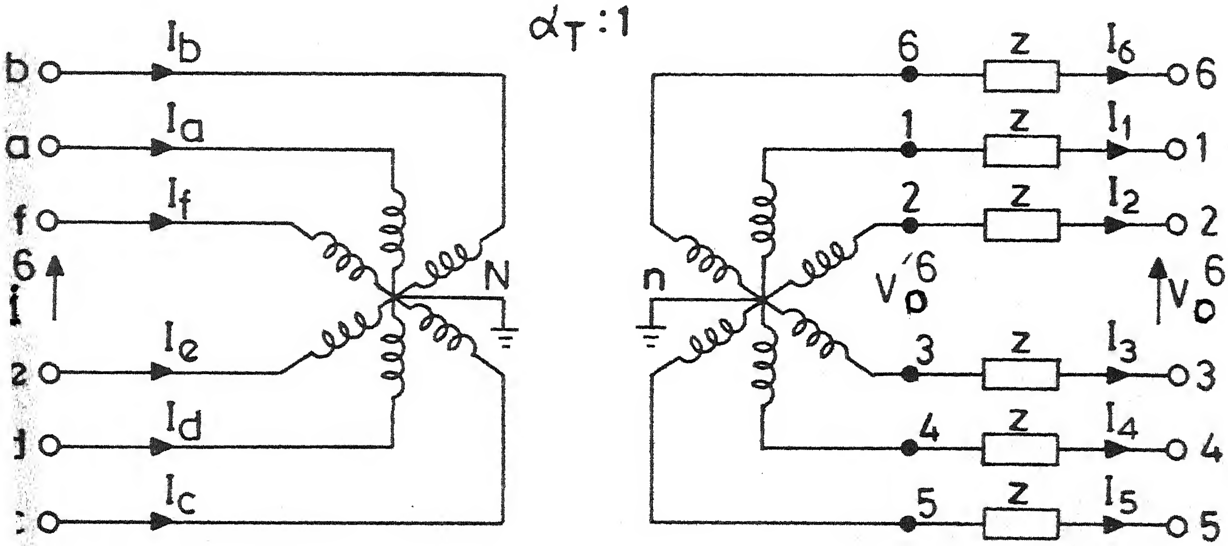


Fig. 2.6 A star/star 6-ϕ transformer of turns ratio α_T:1.

circuit of single-phase units representing parallel windings of multi-phase transformers.

Three-phase/Six-phase transformers

The equivalent circuit representation of three-phase/six-phase transformers utilizes the general symmetrical lattice equivalent circuit (Fig. 2.8) of a single phase transformer (Fig. 2.7) where both primary and secondary windings may have either actual or equivalent variable turns ratio α_T and β_T or both [23]. The parallel transformer windings of three-phase/six-phase transformer are taken to represent equivalent single phase transformer. For example, consider a wye/star transformer (Fig. 2.2) which may be conceptualised as a three winding transformer having :

1) a three-phase winding P with terminal labelled a,b,c,N
2) a secondary equivalent three-phase winding S labelled 1,3,5,n and
3) a tertiary equivalent three-phase winding T labelled 4,6,2,n having 180° phase shift with winding S with the neutrals of S and T joined together.

If y_{PS}, y_{PT} and y_{ST} are short circuit p.u. admittances of the two windings indicated by the subscripts with the third winding open, an equivalent circuit can be assembled by arranging three wye/wye equivalent circuits in parallel in turn as schematically depicted in Fig. 2.9.

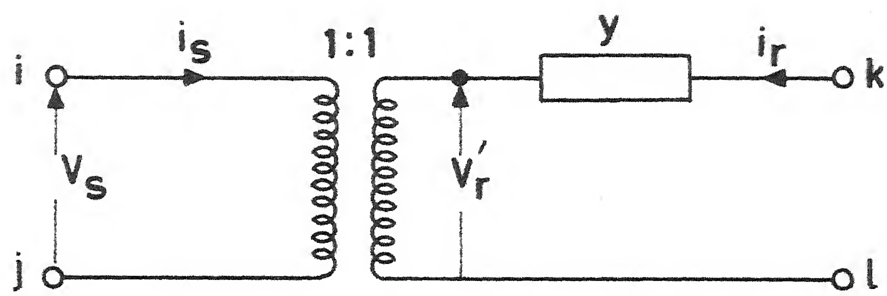


Fig. 2.7 Schematic diagram of a singlephase transformer of turns ratio $\alpha_T : \beta_T$

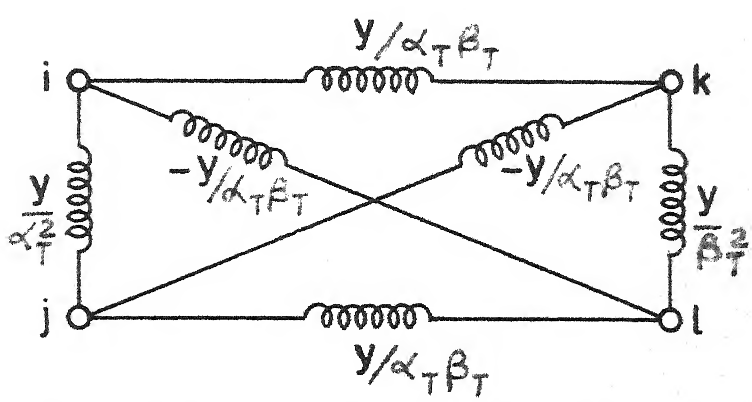


Fig. 2.8 Symmetrical lattice equivalent circuit of the single phase transformer of Fig. 2.7.

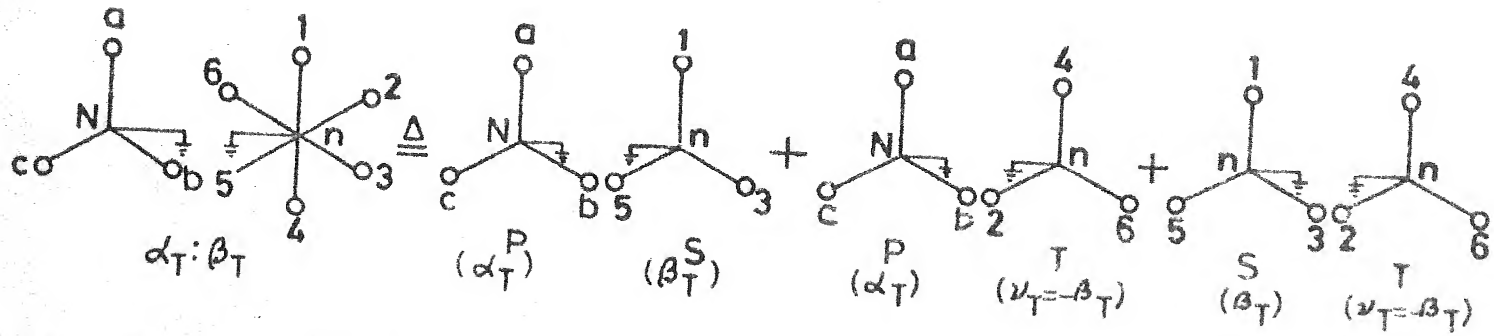


Fig. 2.9 Schematic representation of a wye/star transformer for obtaining an equivalent circuit.

The three windings P,S and T are assumed to be having variable turn ratios α_T, β_T and γ_T respectively. The value of γ_T is taken equal to β_T with negative sign to account for the 180^o phase shift of winding T with respect to S. Matching the parallel sides in the identification of the single phase units a-N with 1-n, a-N with 4-n etc. the connection table showing the admittance connected between different nodes is obtained as given in Table 2.4.

Table 2.4

Connection table for wye/star transformer with equivalent winding P,S and T having the equivalent variable turns ratio $\alpha_T = (1+t_P)$, $\beta = (1+t_S)$ and $\gamma_T = -\beta_T$

Admittance	Between nodes
$(y_{PS} + y_{PT})/\alpha_T^2$	N-a, N-b, N-c
$(y_{PS} + 2y_{ST})/\beta_T^2$	n-1, n-3, n-5
$(y_{PT} + 2y_{ST})/\beta_T^2$	n-4, n-6, n-2
$(y_{PT} - y_{PS})/\alpha_T\beta_T$	n-a, n-b, n-c
$y_{PS}/\alpha_T\beta_T$	a-1, b-3, c-5
$-y_{PS}/\alpha_T\beta_T$	N-1, N-3, N-5
$-y_{PT}/\alpha_T\beta_T$	a-4, b-6, c-2
$y_{PT}/\alpha_T\beta_T$	N-4, N-6, N-2
$-y_{ST}/\beta_T^2$	1-4, 3-6, 5-2

The nodal admittance matrix Y_{TR} of the transformer calculated from Table 2.4 is obtained as,

$$
Y_{TR} = \begin{array}{c|ccc|cccccc|}
 & a & b & c & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
a & x_1 & & & -x_4 & & & x_5 & & \\
b & & x_1 & & & & -x_4 & & & x_5 \\
c & & & x_1 & & x_5 & & & -x_4 & \\
\hline
1 & -x_4 & & & x_2 & & & x_6 & & \\
2 & & & x_5 & & x_3 & & & x_6 & \\
3 & & -x_4 & & & & x_2 & & & x_6 \\
4 & x_5 & & & x_6 & & & x_3 & & \\
5 & & & -x_4 & & x_6 & & & x_2 & \\
6 & & x_5 & & & & x_6 & & & x_3 \\
\hline
\end{array}
\tag{2.15}
$$

where

$$x_1 = (y_{PS} + y_{PT})/\alpha_T^2 \quad ; \quad x_2 = (y_{PS} + y_{PT})/\beta_T^2$$

$$x_3 = (y_{PT} + y_{ST})/\beta_T^2 \quad ; \quad x_4 = y_{PS}/\alpha_T\beta_T$$

$$x_5 = y_{PT}/\alpha_T\beta_T \quad ; \quad x_6 = y_{ST}/\beta_T^2$$

Similarly a delta/star transformer (Fig. 2.3) can be conceptualized as a three winding transformer similar to the wye/star transformer except that in this case the primary is a three-phase delta connected winding with $\alpha_T = \sqrt{3}(1+t_p)$. The equivalent circuit is obtained by paralleling 2 delta/wye and 1 wye/wye equivalent circuits in turn. The connection table 2.5 shows the admittances connected between different

nodes from which the nodal admittance matrix similar to (2.15) can be obtained.

Table 2.5

Connection table for a delta/star transformer with $\alpha_T = \sqrt{3}(1+t_P)$, $\beta_T = (1+t_S)$, $\gamma_T = -\beta_T$.

Admittance	Between nodes
$(y_{PS} + y_{PT})/\alpha_T^2$	a-b, b-c, c-a
$(y_{PS} + 2y_{ST})/\beta_T^2$	1-n, 3-n, 5-n
$(y_{PT} + 2y_{ST})/\beta_T^2$	4-n, 6-n, 2-n
$y_{PS}/\alpha_T\beta_T$	1-c, 3-a, 5-b
$-y_{PS}/\alpha_T\beta_T$	1-b, 3-c, 5-a
$-y_{PT}/\alpha_T\beta_T$	4-c, 6-a, 2-b
$y_{PT}/\alpha_T\beta_T$	4-b, 6-c, 2-a
$-y_{ST}/\beta_T^2$	1-4, 3-6, 5-2

Six Phase Transformer

Six-phase transformer model employing symmetrical lattice equivalent circuit may also be obtained similar to three-phase transformers [23]. For example, a star/star transformer (Fig. 2.6) can be represented by the following nodal admittance matrix derived from the connection table 2.6.

$$Y_{TR} = \begin{array}{c|cc} & a \ \cdots \ f & 1 \ \cdots \ 6 \\ \hline a \\ \vdots & Y_{T1} & Y_{T2} \\ f \\ 1 \\ \vdots & Y_{T2} & Y_{T3} \\ 6 \end{array} \qquad (2.16)$$

where

$$Y_{T1} = \text{Diag.}\ [1\ 1\ 1\ 1\ 1\ 1]\ (y/\alpha_T^2)$$

$$Y_{T2} = \text{Diag.}\ [1\ 1\ 1\ 1\ 1\ 1]\ (-y/\alpha_T\beta_T)$$

$$Y_{T3} = \text{Diag.}\ [1\ 1\ 1\ 1\ 1\ 1]\ (y/\beta_T^2)$$

where y is the leakage admittance in p.u. of the transformer.

Table 2.6

Connection table for a star/star transformer with $\alpha_T = (1+t_p)$ and $\beta_T = (1+t_s)$

Admittance	Between nodes
y/α_T^2	N-a, N-b, N-c, N-d, N-e, N-f
y/β_T^2	n-1, n-2, n-3, n-4, n-5, n-6
$y/\alpha_T\beta_T$	a-1, b-2, c-3, d-4, e-5, f-6
$-y/\alpha_T\beta_T$	n-a, n-b, n-c, n-d, n-e, n-f
$-y/\alpha_T\beta_T$	N-1, N-2, N-3, N-4, N-5, N-6
$6y/(\alpha_T\beta_T)$	N-n

In deriving the above representations, the effect of magnetising core and saturation are neglected. For three-phase/six-phase transformers, a careful examination of models I and II eqns. (2.10) and (2.15) show that the nodal admittance matrices obtained in the two cases are not same. In fact, model II is more precise than model I as it considers the mutual coupling of windings (S and T) which are placed on the same core whereas no such coupling has been considered in Model I. However, the advantage of model I is that it is simple to obtain and is reasonably accurate. For six-phase two winding star/star transformer, the two representations are identical. The above procedure of modelling may be extended to other types of transformer also.

2.3.3 Transmission Lines

Representations of multi-phase line suitable for balanced as well as unbalanced analysis are derived. The description of a multi-phase line in terms of phase impedance/admittance matrix, π circuit and ABCD-parameters is generalised to a n-phase order system. The equivalent three-phase representations of a six-phase line in terms of impedance/admittance matrix and ABCD-parameters employing the realistic models of transformers are developed for carrying out the analysis on an equivalent three-phase basis. Similarly, the six-phase equivalent of a three-phase line is also obtained to carry out the analysis on equivalent six-phase

basis. The equivalent single phase parameters of a six-phase line based on its three-phase representation is also derived.

Phase Impedance, π-circuit and ABCD-parameter Models

An untransposed multi-phase transmission line possessing cyclic symmetry is described by its phase impedance matrix for a general n-phase order as;

$$Z_p^n = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{i1} & Z_{i2} & \cdots & Z_{in} \\ \vdots & & \cdots & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \qquad (2.17)$$

The symmetrical and Clarke's component transformations for higher order systems and particularly six-phase systems are available in literature [14,16,17,21,22] to fully diagonalise the matrix in (2.17).

Whereas the short lines can be represented by (2.17), the adequate representation of medium and long length lines may be obtained by straight forward extension of a nominal and equivalent π circuit representation to multi-phase systems as

$$\begin{bmatrix} I^n_{pS} \\ \\ I^n_{pR} \end{bmatrix} = \begin{bmatrix} Y^n_p + \frac{1}{2} Y^n_{sh} & -Y^n_p \\ \\ -Y^n_p & Y^n_p + \frac{1}{2} Y^n_{sh} \end{bmatrix} \begin{bmatrix} V^n_{pS} \\ \\ V^n_{pR} \end{bmatrix} \tag{2.18}$$

where the second subscripts S and R denote sending and receiving ends respectively. The matrix Y^n_p in (2.18) is related to its symmetrical component matrix Y^n_S similar to (2.2).

Another useful representation, in terms of ABCD-parameters commonly used with several system studies may be written as

$$\begin{bmatrix} V^n_{pS} \\ \\ I^n_{pS} \end{bmatrix} = \begin{bmatrix} A^n & B^n \\ \\ C^n & D^n \end{bmatrix} \begin{bmatrix} V^n_{pR} \\ \\ I^n_{pR} \end{bmatrix} \tag{2.19}$$

where A^n, B^n, C^n and D^n are square matrices of order n.

Equivalent three-phase representation of a six-phase (multi-phase) line

A six-phase (multi-phase) transmission line may be represented by its three-phase equivalent for carrying out the analysis of a composite three-phase and six-phase system. For example, consider a six-phase transmission line connected between three-phase buses S and R via three-phase/six-phase transformers T_1 and T_2 as shown in Fig. 2.10. Let the six-phase line connected between six-phase buses S' and R' be represented as,

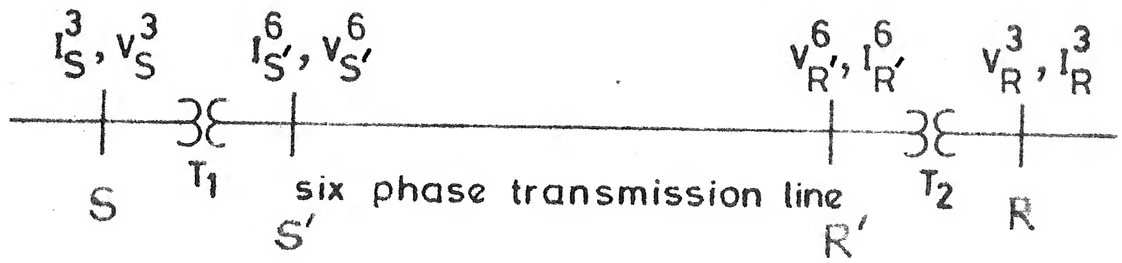


Fig. 2.10 Schematic diagram of a six-phase line connected to 3-ϕ buses S and R via 3-ϕ/6-ϕ transformer.

$$V_{S'}^6 - V_{R'}^6 = Z_p^6\, I_p^6 \tag{2.20}$$

Let the transformers T_1 and T_2 be wye/star, three-phase/six-phase transformers of nominal turns ratios. Then using the voltage relationships of (2.4), the three-phase voltages t bus bars S and R may be expressed in terms of the six-phase voltages $V_{S'}^6$ and $V_{R'}^6$ and the current I_p^6 respectively as

$$V_S^3 = \tfrac{1}{2}\, N^T V_{S'}^6 + \tfrac{1}{2} N^T [z_1] I_p^6 \tag{2.21}$$

and

$$V_R^3 = \tfrac{1}{2}\, N^T V_{R'}^6 - \tfrac{1}{2} N^T [z_2] I_p^6 \tag{2.22}$$

where $[z_1]$ and $[z_2]$ are the diagonal (6x6) matrices representing the leakage impedances of the transformers T_1 and T_2 respectively.

Subtracting eqn. (2.22) from eqn. (2.21) and using eqn. (2.5) yields

$$V_S^3 - V_R^3 = [\tfrac{1}{2} N^T Z_p^6 N + \tfrac{1}{2} N^T \{[z_1] + [z_2]\} N] I_p^3 \tag{2.23}$$

Thus from eqn. (2.23)

$$Z_{p,eq}^3 = \tfrac{1}{2} N^T [Z_p^6 + z_1 + z_2] N \tag{2.24}$$

It may be noted that when the leakage impedances of the transformers are neglected, eqn. (2.24) simplifies to the eqn. (2.25) [15] as shown,

$$Z_{p,eq}^3 = \tfrac{1}{2} N^T Z_p^6 N \tag{2.25}$$

The three-phase equivalent admittance matrix of a six-phase line can either be obtained by inverting $Z^3_{p,eq}$ or derived as follows :

$$Y^3_{p,eq} = [U + \tfrac{1}{2}N^T Y^6_p\{[z_1] + [z_2]\}N]^{-1}\ [\tfrac{1}{2}N^T Y^6_p N] \qquad (2.26)$$

where U is a 3x3 unity matrix.

Again when the leakage impedances of transformers are neglected, eqn. (2.26) simplifies to the eqn. (2.27) [15] as shown,

$$Y^3_{p,eq} = \tfrac{1}{2}\, N^T Y^6_p N \qquad (2.27)$$

Assuming that the six-phase line is balanced such that its phase impedance matrix is cyclic and given by,

$$Z^6_p = \begin{bmatrix} Z_s & Z_{m1} & Z_{m2} & Z_{m3} & Z_{m4} & Z_{m5} \\ Z_{m5} & Z_s & Z_{m1} & Z_{m2} & Z_{m3} & Z_{m4} \\ Z_{m4} & Z_{m5} & Z_s & Z_{m1} & Z_{m2} & Z_{m3} \\ Z_{m3} & Z_{m4} & Z_{m5} & Z_s & Z_{m1} & Z_{m2} \\ Z_{m2} & Z_{m3} & Z_{m4} & Z_{m5} & Z_s & Z_{m1} \\ Z_{m1} & Z_{m2} & Z_{m3} & Z_{m4} & Z_{m5} & Z_s \end{bmatrix} \qquad (2.28)$$

Then the impedance matrix of the equivalent three-phase representation vide eqn . (2.25) is,

It is interesting to note that the equivalent three-phase line of a six-phase fully transposed line is characterised by equal sequence impedances contrary to the case of a normal three-phase line where $Z^{(o)} > Z^{(1)}$ and $Z^{(1)} = Z^{(2)}$.

It may be noted that the equivalent representations given by (2.24) and (2.26) will be different for different connections. However, equations (2.24) and (2.26) may be written in general terms as :

$$Z^3_{p,eq} = k\ H^T[Z^6_p + z_1 + z_2]H \tag{2.32}$$

and

$$Y^3_{p,eq} = [U + k'H^TY^6_p[z_1+z_2]]^{-1}\ [k''\ H^TY^6_p\ H] \tag{2.33}$$

where substitution of various constants k, k' and k" and the matrix H for different transformer connections will yield the required equivalent three-phase impedance and admittance matrices. The constants k, k' and k" for different transformer connections are summarised as under.

- Wye/star and wye/hexagon transformer

  $k = \alpha_T^2/2 \qquad k' = \frac{1}{2} \qquad k'' = 1/(2\alpha_T^2)$

- Delta/star and delta/hexagon transformer

  $k = \alpha_T^2/6 \qquad k' = 1/6 \qquad k'' = 1/(6\alpha_T^2)$

The six-phase line forming a part of the three-phase power system connected at buses S and R via three-phase/six-phase transformers T_1 and T_2 (Fig. 2.10) may be modelled by

equivalent three-phase ABCD-parameters. The terminal relationships at S and S' and R and R' are given by

$$\begin{bmatrix} V^3_{pS} \\ I^3_{pS} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}N^T & \frac{1}{2}N^T[z_1] \\ 0 & \frac{1}{2}N^T \end{bmatrix} \begin{bmatrix} V^6_{pS'} \\ I^6_{pS'} \end{bmatrix} \tag{2.34}$$

and

$$\begin{bmatrix} V^6_{pR'} \\ I^6_{pR'} \end{bmatrix} = \begin{bmatrix} N & [z_2]N \\ 0 & N \end{bmatrix} \begin{bmatrix} V^3_{pR} \\ I^3_{pR} \end{bmatrix} \tag{2.35}$$

From (2.34), (2.35) and (2.19) with n = 6, the terminal quantities of bus-bars S and R can be related as

$$\begin{bmatrix} V^3_{pS} \\ I^3_{pS} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}N^T & \frac{1}{2}N^T[z_1] \\ 0 & \frac{1}{2}N^T \end{bmatrix} \begin{bmatrix} A^6 & B^6 \\ C^6 & D^6 \end{bmatrix} \begin{bmatrix} N & [z_2]N \\ 0 & N \end{bmatrix} \begin{bmatrix} V^3_{pR} \\ I^3_{pR} \end{bmatrix} \tag{2.35}$$

which can be written as

$$\begin{bmatrix} V^3_{pS} \\ I^3_{pS} \end{bmatrix} = \begin{bmatrix} A^3_{eq} & B^3_{eq} \\ C^3_{eq} & D^3_{eq} \end{bmatrix} \begin{bmatrix} V^3_{pR} \\ I^3_{pR} \end{bmatrix} \tag{2.36}$$

In view of the fact that $A^6 = D^6$, and A,B,C,D, $[z_1]$ and $[z_2]$ are diagonal matrices, eqn. (2.36) after simplification yields,

$$A^3_{eq} = \tfrac{1}{2}N^T A^6 N + \tfrac{1}{2}N^T[z_1]C^6 N$$

$$B^3_{eq} = \tfrac{1}{2}N^T B^6 N + \tfrac{1}{2}N^T A^6[z_1+z_2]N + \tfrac{1}{2}N^T[z_1]C^6[z_2]N \qquad (2.37)$$

$$C^3_{eq} = \tfrac{1}{2} N^T C^6 N$$

$$D^3_{eq} = \tfrac{1}{2}N^T D^6 N + \tfrac{1}{2}N^T C^6[z_2]N$$

Note that the equivalent three-phase parameters A^3_{eq} and D^3_{eq} are no longer equal, except when $[z_1] = [z_2]$. In the special case when leakage impedances of T_1 and T_2 are neglected, eqn. (2.37) simplifies to the eqn. (2.38) [15].

$$\begin{aligned} A^3_{eq} &= \tfrac{1}{2} N^T A^6 N \\ B^3_{eq} &= \tfrac{1}{2} N^T B^6 N \\ C^3_{eq} &= \tfrac{1}{2} N^T C^6 N \\ D^3_{eq} &= \tfrac{1}{2} N^T D^6 N \end{aligned} \qquad (2.38)$$

Equation (2.37) can be put in general terms from which the ABCD-parameters for different transformer connections can be obtained by putting respective values of constant k and matrix H as

$$\begin{aligned} A^3_{eq} &= kH^T [A^6 + [z_1]C^6]H \\ B^3_{eq} &= kH^T[B^6 + A^6[z_1+z_2] + [z_1] C^6 [z_2]]H \\ C^3_{eq} &= kH^T C^6 H \\ D^3_{eq} &= kH^T[D^6 + C^6[z_2]]H \end{aligned} \qquad (2.39)$$

For investigation of the problem of unbalances in phase coordinates, the representation of interfacing three-phase/six-phase transformers by their symmetrical lattice equivalent to derive equivalent three-phase model of six-phase transmission line may be used. For example, consider the six-phase transmission line of Fig. 2.10 and let the nodal admittance description of transformer T_1, line, and transformer T_2 be written as,

$$\begin{bmatrix} I_S^3 \\ -I_{S'}^6 \end{bmatrix} = \begin{bmatrix} Y_{TR1}^{(1)} & Y_{TR2}^{(1)} \\ Y_{TR3}^{(1)} & Y_{TR4}^{(1)} \end{bmatrix} \begin{bmatrix} V_S^3 \\ V_{S'}^6 \end{bmatrix} \tag{2.40}$$

$$\begin{bmatrix} I_{S'}^6 \\ -I_R^6 \end{bmatrix} = \begin{bmatrix} Y_{TL1} & Y_{TL2} \\ Y_{TL3} & Y_{TL4} \end{bmatrix} \begin{bmatrix} V_{S'}^6 \\ V_{R'}^6 \end{bmatrix} \tag{2.41}$$

and

$$\begin{bmatrix} I_{R'}^6 \\ -I_R^3 \end{bmatrix} = \begin{bmatrix} Y_{TR1}^{(2)} & Y_{TR2}^{(2)} \\ Y_{TR3}^{(2)} & Y_{TR4}^{(2)} \end{bmatrix} \begin{bmatrix} V_{R'}^6 \\ V_R^3 \end{bmatrix} \tag{2.42}$$

Elimination of intermediate nodes corresponding to six-phase buses S' and R' by the use of eqns. (2.40) - (2.42) leads to the relationships between currents and voltages of three-phase buses S and R as,

$$\begin{bmatrix} I_S^3 \\ \\ -I_R^3 \end{bmatrix} = \begin{bmatrix} \\ Y_{p,eq}^3 \\ \\ \end{bmatrix} \begin{bmatrix} V_S^3 \\ \\ V_R^3 \end{bmatrix} \tag{2.43}$$

The desired equivalent three-phase nodal admittance matrix follows from (2.43) as

$$Y_{p,eq}^3 = \begin{bmatrix} Y_{TR1}^{(1)} - Y_{TR2}^{(1)} X_1 Y_{TR3}^{(1)} & -Y_{TR2}^{(2)} X_2 Y_{TR3}^{(2)} \\ \\ -Y_{TR1}^{(2)} X_3 Y_{TR3}^{(1)} & Y_{TR1}^{(2)} - Y_{TR1}^{(2)} X_4 Y_{TR4}^{(2)} \end{bmatrix} \tag{2.44}$$

where

$$\begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix} = \begin{bmatrix} Y_{TR4}^{(1)} + Y_{TL1} & Y_{TR2}^{(1)} \\ \\ Y_{TL3} & Y_{TL4} + Y_{TR4}^{(2)} \end{bmatrix}^{-1} \tag{2.45}$$

Alternatively, the nodal admittance matrix of the network between buses S and R may be assembled as,

$$Y = \begin{array}{|c|c|c|c|} \hline Y_{TR1}^{(1)} & Y_{TR2}^{(1)} & & \\ \hline Y_{TR3}^{(1)} & Y_{TR4}^{(1)}+Y_{TL1} & Y_{TL2} & \\ \hline & Y_{TL3} & Y_{TL4}+Y_{TR1}^{(2)} & Y_{TR2}^{(2)} \\ \hline & & Y_{TR3}^{(2)} & Y_{TR4}^{(2)} \\ \hline \end{array} \tag{2.46}$$

and the intermediate nodes corresponding to S' and R' may be eliminated by usual node elimination subroutine. The

equivalent three-phase nodal admittance matrix once calculated may be used as and when required for subsequent analyses.

Equivalent Single phase representation based on three-phase description of system

A single phase equivalent of a six-phase line for the analysis of the system entirely on three-phase basis may be conveniently derived from the equivalent three-phase ABCD-parameters. Note that A^3_{eq}, B^3_{eq}, C^3_{eq} and D^3_{eq} are square diagonal matrices with A^1_{eq}, B^1_{eq}, C^1_{eq} and D^1_{eq} as the diagonal entries respectively. Let $Z^1_{p,eq}$ and $Y^1_{sh,eq}$, and Z^1_p and Y^1_{sh} are the equivalent single phase and per phase series impedance and shunt admittance of the six-phase line respectively, then for a π circuit representation, the following familiar relations can be written

$$A^1_{eq} = 1 + 0.5\, Z^1_{p,eq}\, Y^1_{sh,eq} \tag{2.47}$$

$$B^1_{eq} = Z^1_{p,eq} \tag{2.48}$$

Equating the values of A^1_{eq} and B^1_{eq} from (2.39) with (2.47) and (2.48) respectively, the $Z^1_{p,eq}$ and $Y^1_{p,eq}$ can be related to Z^1_p and Y^1_{sh}. As an illustration, for identical wye/star transformers T_1 and T_2 connected at both ends of the line (Fig. 2.10), the following relations are obtained

$$Z^1_{p,eq} = Z^1_p + 2z(1+0.5Z^1_p Y^1_{sh}) + z^2 Y^1_{sh}(1+0.25Z^1_p Y^1_{sh}) \tag{2.49}$$

$$Y^1_{sh,eq} = \frac{1}{Z^1_{p,eq}}[Z^1_p Y^1_{sh} + 2zY^1_{sh}(1+0.25Z^1_p Y^1_{sh})] \tag{2.50}$$

Equivalent six-phase representation of a three-phase line

In a composite three-phase and six-phase system when the interest of investigation lies in six-phase part of the network, it may be desirable to analyse the system entirely on six-phase basis. For this purpose, a six-phase equivalent of three-phase line network may be derived. For example, consider a three-phase line connected to the six-phase buses via six-phase/three-phase transformer T_1 and T_2 as shown in Fig. 2.11. Employing the terminal relations of the wye/star transformer (eqns. 2.4 - 2.6), and the description of the line as

$$V^3_{S'} - V^3_{R'} = Z^3_p \; I^3_p \; , \tag{2.51}$$

the voltage drop vector $V^6_S - V^6_R$ is obtained as

$$V^6_S - V^6_R = [\tfrac{1}{2} \, N \, Z^3_p N^T + [z_1+z_2]]I^6_p \tag{2.52}$$

Thus the six-phase equivalent representation of the line from (2.52) is given by,

$$Z^6_{p,eq} = \tfrac{1}{2} \, N \, Z^3_p \, N^T + [z_1+z_2] \tag{2.53}$$

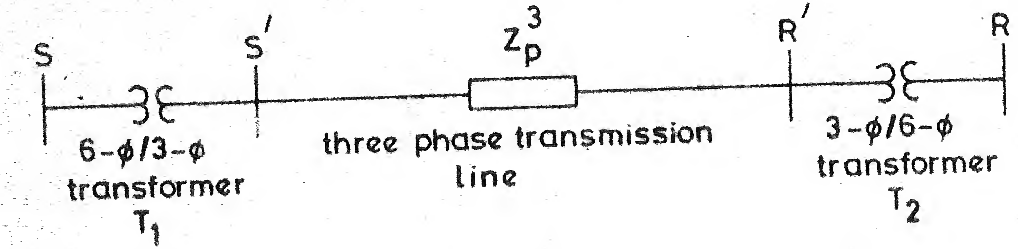


FIG. 2.11 SCHEMATIC DIAGRAM OF A 3-PHASE LINE CONNECTED TO SIX-PHASE BUSES S AND R VIA 3-φ/6-φ TRANSFORMERS T_1 AND T_2

Similarly, the six-phase equivalent admittance matrix is either obtained by inverting $Z^6_{p,eq}$ or can be derived to yield

$$Y^6_{p,eq} = [U+\tfrac{1}{2}NY^3_pN^T[z_1+z_2]]^{-1}\ \tfrac{1}{2}NY^3_pN^T \tag{2.54}$$

where U is (6x6) order unity matrix.

It may be noted that when the leakage impedances of transformers are neglected, eqns. (2.53) and (2.54) simplify to the form

$$Z^6_{p,eq} = \tfrac{1}{2}NZ^3_pN^T \tag{2.55}$$

$$Y^6_{p,eq} = \tfrac{1}{2}NY^3_pN^T \tag{2.56}$$

Assuming that the three-phase line is balanced such that its phase impedance matrix is cyclic and is given by

$$Z^3_p = \begin{bmatrix} Z_S & Z_{m1} & Z_{m2} \\ Z_{m2} & Z_S & Z_{m1} \\ Z_{m1} & Z_{m2} & Z_S \end{bmatrix} \tag{2.57}$$

then the impedance matrix of the equivalent six-phase representation vide eqn. (2.53) is given by

$$Z^6_{p,eq} = \frac{1}{2}\begin{bmatrix} Z_S+z_1+z_2 & -Z_{m2} & Z_{m1} & -Z_S & Z_{m2} & -Z_{m1} \\ -Z_{m1} & Z_S+z_1+z_2 & -Z_{m2} & Z_{m1} & -Z_S & Z_{m2} \\ Z_{m2} & -Z_{m1} & Z_S+z_1+z_2 & -Z_{m2} & Z_{m1} & -Z_S \\ -Z_S & Z_{m2} & -Z_{m1} & Z_S+z_1+z_2 & -Z_{m2} & Z_{m1} \\ Z_{m1} & -Z_S & Z_{m2} & -Z_{m1} & Z_S+z_1+z_2 & -Z_{m2} \\ -Z_{m2} & Z_{m1} & -Z_S & Z_{m2} & -Z_{m1} & Z_S+z_1+z_2 \end{bmatrix} \tag{2.58}$$

A careful examination of eqn. (2.58) reveals that, the matrix has cyclic symmetry and is non-singular. When leakage impedances are neglected, the resulting matrix becomes singular as obtained in [15] implying that the corresponding admittance matrix does not exist. The symmetrical component transformation of (2.58) yields

$$\begin{aligned} Z^{(o)}_{p,eq} &= z_1+z_2 \\ Z^{(1)}_{p,eq} &= Z_S-\alpha Z_{m1}-\alpha^* Z_{m2} \\ Z^{(2)}_{p,eq} &= z_1+z_2 \\ Z^{(3)}_{p,eq} &= Z_S+Z_{m1}+Z_{m2} \\ Z^{(4)}_{p,eq} &= z_1+z_2 \\ Z^{(5)}_{p,eq} &= Z_S-\alpha^* Z_{m1}-\alpha Z_{m2} \end{aligned} \tag{2.59}$$

where $\alpha = \exp(j2\pi/6) = 0.5+j0.866$ $\alpha^* = 0.5-j0.866$.

For a completely transposed three-phase line employing $Z_{m1} = Z_{m2} = Z_m$, the equivalent six-phase sequence impedances become

$$Z^{(o)}_{p,eq} = Z^{(2)}_{p,eq} = Z^{(4)}_{p,eq} = z_1+z_2$$

$$Z^{(1)}_{p,eq} = Z^{(5)}_{p,eq} = Z_S - Z_m$$

$$Z^{(3)}_{p,eq} = Z_S + 2Z_m \qquad (2.60)$$

It is interesting to note that the zero, second and fourth sequence component become equal to the total leakage impedance of terminal transformers, first and fifth components become equal to the positive sequence impedance of the three-phase line and the third sequence component equals the original zero sequence impedance of the line. For negligible transformer leakage impedances, the equivalent six-phase line is described only by the first, third and fifth sequence components only with the rest of the components reduced to zero value.

It may be noted again that the equivalent six-phase representation given by (2.53) and (2.54) will be different for different transformer connections. However, they may be written in general terms as :

$$Z^6_{p,eq} = k'' H Z^3_p H^T + z_1 + z_2 \qquad (2.61)$$

$$Y^6_{p,eq} = [U+k H Y^3_p H^T [z_1+z_2]]^{-1} k H Y^3_p H^T \qquad (2.62)$$

where substitution of various constants k, k" and matrix H for different connection schemes will yield the required equivalent six-phase impedance and admittance matrices.

The six-phase equivalent ABCD-parameters may also be derived following the procedure similar to that used for deriving three-phase equivalent ABCD-parameters. If A^3, B^3, C^3 and D^3 are the ABCD-parameters of three-phase lines, then the six-phase equivalent parameters are given in general form valid for different transformer schemes as :

$$\begin{aligned}
A^6_{eq} &= k' H A^3 H^T + k[z_1] H C H^T \\
B^6_{eq} &= k'' H B^3 H^T + k' H A^3 H^T [z_2] + k'[z_1] H D^3 H^T + k[z_1] H C^3 H^T [z_2] \\
C^6_{eq} &= k\ H C^3 H^T \qquad (2.63) \\
D^6_{eq} &= k'\ H D^3 H^T + k H C^3 H^T [z_2]
\end{aligned}$$

Again for negligible transformer leakage impedances, the equivalent six-phase ABCD-parameters in (2.63) simplify to the following :

$$\begin{aligned}
A^6_{eq} &= k'\ H A^3 H^T \\
B^6_{eq} &= k''\ H B^3 H^T \qquad (2.64) \\
C^6_{eq} &= k\ H C^3 H^T \\
D^6_{eq} &= k'\ H D^3 H^T
\end{aligned}$$

A more rigorous representation suitable for unbalanced network analysis, in phase coordinates, employing transformer models II and the π circuit model of transmission line may be obtained in the form as given below, if required.

$$\begin{bmatrix} I_p^6 \\ \\ I_R^6 \end{bmatrix} = \begin{bmatrix} & & \\ & Y^6_{p,eq} & \\ & & \end{bmatrix} \begin{bmatrix} V_S^6 \\ \\ V_R^6 \end{bmatrix} \tag{2.65}$$

The procedure is similar to that used in deriving eqn. (2.43).

2.3.4 Multi-phase Loads

Loads on a six-phase (multi-phase) system may be represented by constant impedances/admittances in balanced as well as in unbalanced conditions, as the case may be. The load impedance/admittance matrices may be constructed, for example, from the knowledge of their symmetrical component values employing eqn. (2.2). Loads in the form of machine, if any, may be adequately represented by eqn. (2.1).

As pointed out earlier, for the analysis of phenomena on three-phase part of a composite three-phase and six-phase network such as calculation of fault currents and load flows, it is useful to replace the six-phase element (load) by an equivalent three-phase element [15]. Consider, for example, a six-phase load connected via a three-phase/six-phase transformer to three-phase bus as shown in Fig. 2.12. Recalling the voltage and current relationships of transformer

eqns. (2.4) and (2.5), and representation of the six-phase load as

$$V_p^6 = Z_p^6 \, I_p^6 \tag{2.66}$$

The voltage V_p^3 can be written as

$$V_p^3 = \tfrac{1}{2}N^T Z_L^6 N I_p^3 + \tfrac{1}{2}N^T [z] N I_p^3 \tag{2.67}$$

From (2.67) the equivalent three-phase load is obtained as

$$Z_{L,eq}^3 = \tfrac{1}{2}N^T [Z_L^6 + z] N \tag{2.68}$$

Similarly, the equivalent three-phase admittance matrix of the six-phase admittance load Y_L^6 can be obtained either by inverting $Z_{L,eq}^3$ or can be derived to be

$$Y_{L,eq}^3 = [U + \tfrac{1}{2}N^T Y_L^6 [z] N]^{-1} \, [\tfrac{1}{2}N^T Y_L^6 N] \tag{2.69}$$

In the special case where leakage impedance of interfacing transformer is neglected, the equivalent representations eqns. (2.68) and (2.69) simplify to the form [15]

$$Z_{L,eq}^3 = \tfrac{1}{2}N^T Z_L^6 N \tag{2.70}$$

$$Y_{L,eq}^3 = \tfrac{1}{2}N^T Y_L^6 N \tag{2.71}$$

Further, eqns. (2.68) and (2.69) may be put in general terms as

$$Z_{L,eq}^3 = k \, H^T [Z_L^6 + z] H \tag{2.72}$$

and

$$Y^3_{L,eq} = [U + k' H^T Y^6_L [z] H]^{-1} \; [k'' \; H^T Y^6_p H] \qquad (2.73)$$

and by substituting appropriate values of constant k, k' and k", and matrix H will yield the respective representation valid for transformer connection scheme of interest.

One of the possible schemes to tap a six-phase line at some location would be to employ a six-phase/three-phase transformer for the purpose. Consider a three-phase load connected to a six-phase bus via a six-phase/three-phase, star/wye transformer (Fig. 2.13). Using the terminal relations of transformer eqns. (2.4) and (2.5), and the load representation as

$$V^3_p = Z^3_L \; I^3_p \qquad (2.74)$$

the voltage V^6_p can be written as

$$V^6_p = \tfrac{1}{2} N Z^3_L N^T I^6_p + [z] I^6_p \qquad (2.75)$$

The equivalent six-phase load as obtained from (2.75) is as

$$Z^6_{L,eq} = \tfrac{1}{2} N Z^3_L N^T + z \qquad (2.76)$$

Similarly, the equivalent representation in admittance form is obtained as

$$Y^6_{L,eq} = [U + \tfrac{1}{2} N Y^3_L N^T z]^{-1} \; [\tfrac{1}{2} N Y^3_L N^T] \qquad (2.77)$$

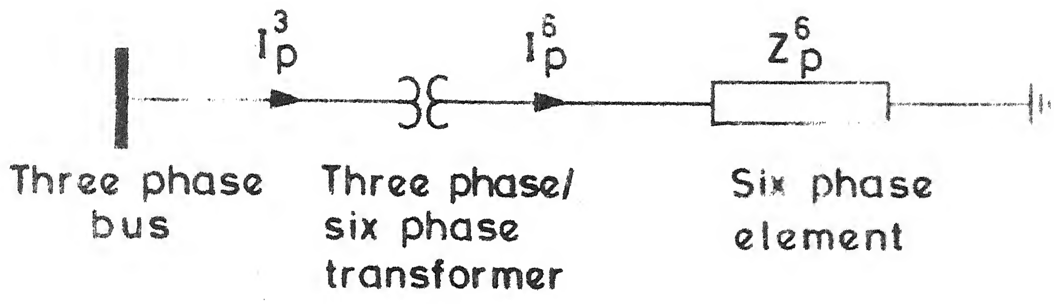


FIG. 2.12 A SIX-PHASE (MULTI-PHASE) LOAD CONNECTED TO THREE-PHASE BUS BARS

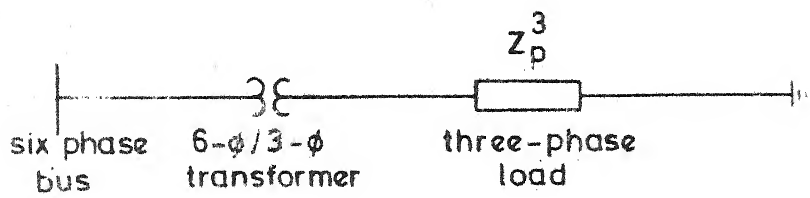


FIG. 2.13 A THREE-PHASE LOAD TAPPED FROM A SIX-PHASE LINE OR BUS

In the special case when the transformer leakage impedances are neglected, the eqns. (2.76) and (2.77) simplify to the form [15]

$$Z^6_{L,eq} = \tfrac{1}{2} N Z^3_L N^T \tag{2.78}$$

$$Y^6_{L,eq} = \tfrac{1}{2} N Y^3_L N^T \tag{2.79}$$

Further the eqns. (2.76) and (2.77) can be cast in general format as

$$Z^6_{L,eq} = [k_4 \; H \; Z^3_L \; H^T + z] \tag{2.80}$$

and

$$Y^6_{L,eq} = [U + k_5 H Y^3_L H^T z]^{-1} \; [k_5 H Y^3_L H^T] \tag{2.81}$$

where the values of constant k_4 and k_5 for different transformer connections are as follows :

- Star/wye and hexagon/wye transformer

$$k_4 = 1/(2\alpha_T^2) \qquad k_5 = \alpha_T^2/2$$

- Star/delta and hexagon/delta transformer

$$k_4 = 1/(6\alpha_T^2) \qquad k_5 = \alpha_T^2/6 \; .$$

2.4 DISCUSSION

In first half of the chapter, an overview of feasibility of multi-phase systems has been discussed based on the studies reported in the literature [2-8]. An attempt has been

made to focus the salient features of performance characteristics of multi-phase systems in comparison to conventional three-phase systems. In addition, the advances made towards construction of experimental lines, their testing and design capabilities, and the areas where multi-phase system may find future applications are detailed.

In the second half of the chapter, modelling of various elements of a multi-phase system suitable for steady state analysis has been presented. Three-phase/six-phase transformers which are needed to link the multi-phase elements with the rest of the network have been modelled in two alternative ways. Model I is simple to obtain and reasonably accurate and has been extensively employed to derive several useful results. Model II is more precise than Model I and is suitable for rigorous representation of the overall system intended for unbalanced network analysis in phase coordinates. A six-phase transformer is also treated in both ways and it is shown that the two representations are identical. Transmission line models for short, medium and longlength are also given. The equivalent three-phase representation of a multi-phase line and the multi-phase equivalent of a three-phase line with respect to impedance/admittance matrix and ABCD-parameters are developed. The symmetrical component transformations of equivalent three-phase and six-phase line are discussed and are shown to possess sequence values of impedances which are

entirely different in magnitude as compared to the normal three-phase and six-phase lines. The representation of a six-phase generator which may become a reality in future, is developed incorporating the imbalances in the machine particularly suitable for unbalanced analysis. The various representations of loads are detailed to include : three-phase equivalent of a multi-phase load, multi-phase equivalent of a three-phase load, and the constant impedance/admittance description both for balanced as well as unbalanced conditions. In addition, the single phase equivalent of the six-phase transmission line has also been derived. These element models are used in the following chapter to describe the overall system for balanced as well as unbalanced system studies.

CHAPTER III

LOAD FLOW ANALYSIS

3.1 INTRODUCTION

This chapter is devoted to the investigation of load flow problems of six-phase (multi-phase) power system which has not been explored as yet except the balanced single phase load flow by Venkata et al [2]. Firstly, in the Section 3.2, the representations of overall system which are complicated by the interconnections of various elements having different number of phases through interfacing transformers, are developed. The balanced system analysis of such complex networks requires representation of the overall system either on equivalent three-phase or six-phase basis so that the positive sequence network derived therefrom may be used for equivalent single phase study. However, the phase coordinate method affords a very powerful and flexible tool to model such networks and allows the representation of unbalances as they exist in the system. Employing the models of six-phase (multi-phase) elements presented in Chapter 2 along with the usual three-phase elements [23-31,37] in phase coordinates, the procedure for assembling a multi-phase bus admittance matrix is developed in Section 3.2.4.

Employing these modelling schemes of multi-phase system, various alternative schemes for load flow studies depending upon the nature and the point of interest of investigation in three phase, or six-phase or both parts of the network are presented for balanced as well as unbalanced conditions in Section 3.3. The validity of the proposed schemes is demonstrated by working out a sample system. The load flow schemes presented in this chapter are utilized to work out various case studies including the impact of converting double circuit three-phase lines to six-phase lines in Section 3.4. In addition, the problem of obtaining effective starting solutions (especially for a large power system network) is also investigated and their computational requirements for various cases are quantified. Finally, in the Section 3.5, the chapter concludes with discussion of salient features of the study.

3.2 SYSTEM REPRESENTATION

The models of six-phase (multi-phase) elements developed in Chapter 2 along with those of conventional three-phase elements [23-31,37] are employed to represent a multi-phase or composite three-phase and six-phase system : on equivalent single-phase basis for balanced analysis and in phase coordinates alternatively on; equivalent three-phase, six-phase, and mixed three-phase and six-phase basis particularly suitable for unbalanced analyses. In each case, the system may be represented in the form;

$$YV = I \tag{3.1}$$

relating the nodal voltages V and currents I. The matrix Y in (3.1) is called the base case system nodal admittance matrix. The procedure for obtaining (3.1) in various alternative ways are discussed separately and illustrated with reference to a sample system of Fig. 3.1 with the data marked on the diagram.

3.2.1 Equivalent Single Phase Representation

The balanced system analyses are usually carried out employing the positive sequence network descriptions. The derivation of positive sequence network for a completely six-phase system presents no difficulty. However, because of the presence of elements having different phase orders including the interfacing transformers, the derivation of positive sequence network for a composite three-phase and six phase system is not straightforward. There are two options available;

(i) to represent the entire system on three phase basis, or
(ii) to represent the entire system on six-phase basis.

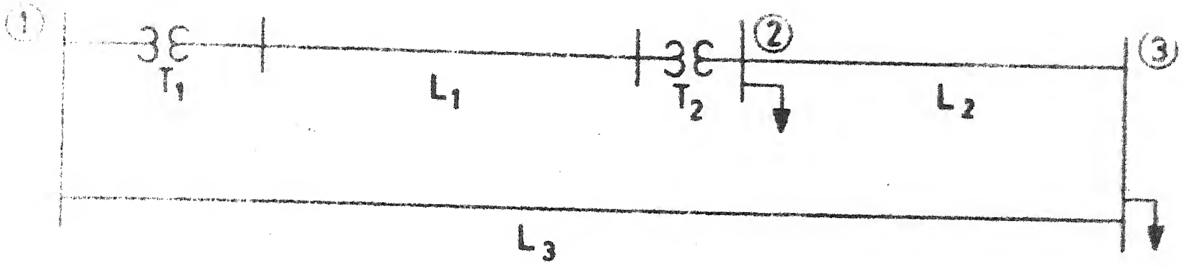
Once the system is represented employing the equivalent models (detailed in Chapter 2), either by option (i) or (ii), the calculation of positive sequence network is simple.

To illustrate the procedure, the sample system of Fig. 3.1 is worked out for both the options and the various steps involved are detailed in Appendix ~~B~~.

3.2.2 Equivalent three-phase (or six-phase) representation

To illustrate the three-phase representation, the sample system of Fig. 3.1 is redrawn in Fig. 3.2. showing the bus numbering sequence. The interconnection of the network elements is shown along with their nodal admittance matrices in Table 3.1. The three-phase nodal admittance matrix of the system can be assembled [25] with the help of connection table as,

$$\begin{array}{c} \\ 1 \\ \vdots \\ 3 \\ 4 \\ \vdots \\ 6 \\ 7 \\ \vdots \\ 9 \end{array}
\begin{array}{c}
\begin{array}{ccc} 1 \cdots 3 & 4 \cdots 6 & 7 \cdots 9 \end{array} \\
\left[\begin{array}{c|c|c}
Y^3_{L1}+Y^3_{L3} & -Y^3_{L1} & -Y^3_{L3} \\ \hline
-Y^3_{L1} & Y^3_{L1}+Y^3_{L2} & -Y^3_{L2} \\ \hline
-Y^3_{L3} & -Y^3_{L2} & Y^3_{L2}+Y^3_{L3}
\end{array}\right]
\end{array}
\qquad (3.2)$$



SIX-PHASE TRANSMISSION LINE L1 : $Z_o = j0.45$, $Z_1 = Z_2 = Z_3 = Z_4 = Z_5 = j0.08$

THREE PHASE LINES L2 AND L3 : $Z_o = j0.25$, $Z_1 = Z_2 = j0.08$

THREE-PHASE/SIX-PHASE, WYE/STAR TRANSFORMERS T_1 AND T_2 : $Z_o = Z_1 = Z_2 = j0.10$

$y_{PS} = y_{PT} = y_{ST} = -j20.0$

FIG. 3.1 SINGLE LINE DIAGRAM OF A MULTI-PHASE POWER SYSTEM NETWORK

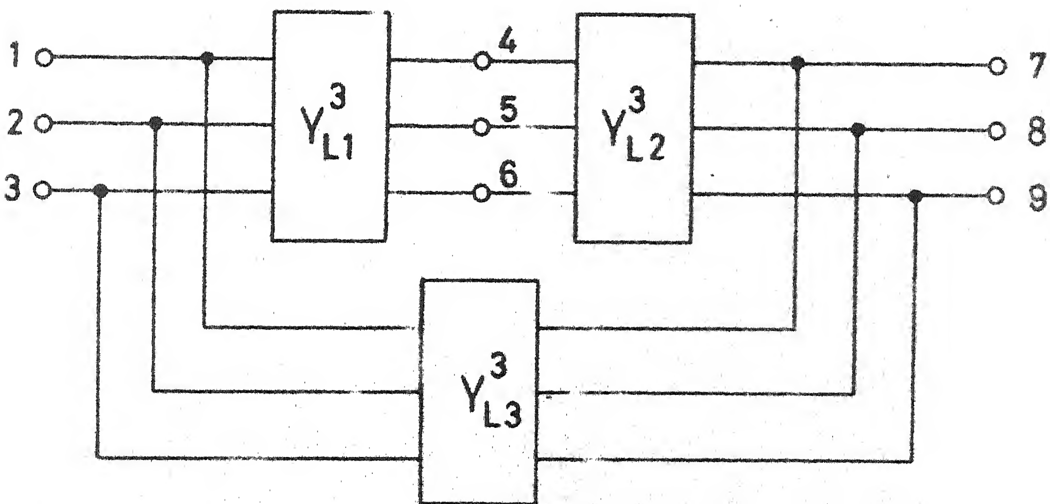


FIG. 3.2 SCHEMATIC DIAGRAM OF SAMPLE SYSTEM OF FIG. 3.1 SHOWING BUS NUMBERING SEQUENCE FOR THREE-PHASE REPRESENTATION

Table 3.1

Three phase connection table for network in Fig. 3.1

Terminating bus groups N_1	N_2	Three-phase Branch Admittance Matrix
1,2,3	4,5,6	Y^3_{L1}
4,5,6	7,8,9	Y^3_{L2}
1,2,3	7,8,9	Y^3_{L3}

Substituting the numerical values of Y^2_{L1}, Y^3_{L2} and Y^3_{L3} from eqns. (B.1) and (B.5) in Appendix B the (9x9) order nodal admittance matrix is given by,

$Y = jX$

1	2	3	4	5	6	7	8	9
-13.625	2.833	2.833	3.959			9.666	-2.833	-2.833
2.833	-13.625	2.833		3.959		-2.833	9.66	-2.833
2.833	2.833	-13.625			3.959	-2.833	-2.833	9.666
3.959			-13.625	2.833	2.833	9.666	-2.833	-2.833
	3.959		2.833	-13.625	2.833	-2.833	9.666	-2.833
		3.959	2.833	2.833	-13.625	-2.833	-2.833	9.666
9.666	-2.833	-2.833	9.666	-2.833	-2.833	-19.332	5.666	5.666
-2.833	9.666	-2.833	-2.833	9.666	-2.833	5.666	-19.332	5.666
-2.833	-2.833	9.666	-2.833	-2.833	9.666	5.666	5.666	-19.332

(3.3)

In a similar manner, the equivalent six-phase representation of the overall system employing six-phase equivalent models of three-phase elements, can be obtained.

3.2.3 Mixed three-phase and six-phase representation

To visualize the mixed three-phase and six-phase representation retaining the physical identities of various elements, the sample system of Fig. 3.1 (with a three-phase generator included at bus 1) is redrawn in Fig. 3.3 showing the bus numbering sequence. Inspection of Fig. 3.3 yields the multi-phase connection table 3.2 showing the connection of various elements and their nodal admittance sub-matrices between the various bus bar groups. The assembly of the system nodal admittance matrix is carried out in modular fashion and by taking one element at a time. The procedure is illustrated as under.

3.2.4 The assembly of system nodal admittance matrix

Step 1 : The nodal admittance sub-matrix of element 1 which is a three-phase generator is calculated employing phase values or sequence component values making use of the relation

$$Y_G^3 = (T_S^3)(Y_{comp}^3)(T_S^{3*}) \tag{3.4}$$

where Y_{comp}^3 is the symmetrical component admittance matrix and T_S^3 is the symmetrical component transformation matrix for three-phase system.

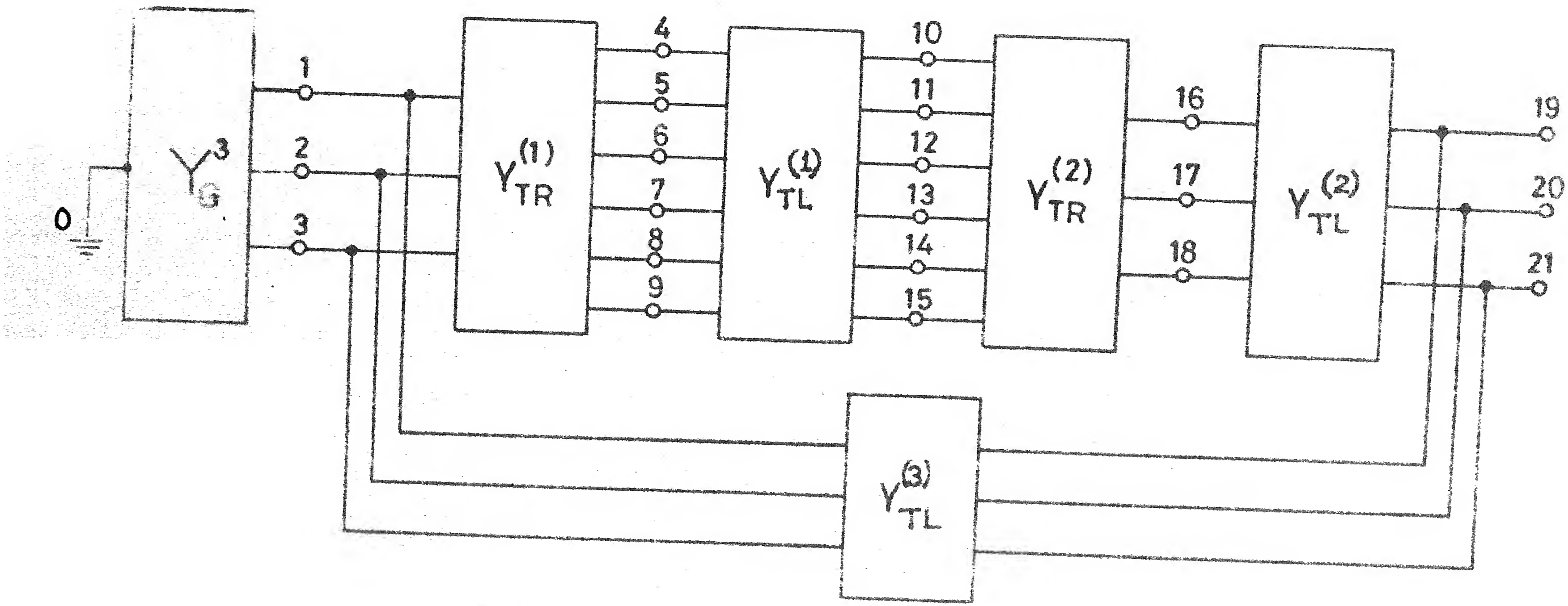


FIG. 3.3 SCHEMATIC DIAGRAM OF SAMPLE SYSTEM OF FIG. 3.1 SHOWING BUS NUMBERING SEQUENCE FOR MIXED THREE-PHASE AND SIX-PHASE REPRESENTATION

Table 3.2

Multi-phase connection table of network in Fig. 3.1

S.No.	Element	Terminating bus bars N1	Terminating bus bars N2	Nodal admittance matrix	Remarks
1	Three-phase generator G	1-3	0	Y_G^3	Phase admittance matrix of order (3x3)
2	Wye/star transformer T_1	1-3	4-9	$Y_{TR}^{(1)}$	With submatrices $Y_{TR1}^{(1)}$, $Y_{TR2}^{(1)}$, $Y_{TR3}^{(1)}$ and $Y_{TR4}^{(1)}$ of dimensions (3x3),(3x6),(6x3) and (6x6) respectively
3	Three phase line L3	1-3	19-21	$Y_{TL}^{(3)}$	With submatrices Y_{L3}^3, $-Y_{L3}^3$, $-Y_{L3}^3$ and Y_{L3}^3 all of dimension (3x3)
4	Six-phase line L1	4-9	10-15	$Y_{TL}^{(1)}$	With submatrices Y_{L1}^6, $-Y_{L1}^6$, $-Y_{L1}^6$ and Y_{L1}^6 all of dimension(6x6)
5	Star/wye transformer T_2	10-15	16-18	$Y_{TR}^{(2)}$	With submatrices $Y_{TR1}^{(2)}$, $Y_{TR2}^{(2)}$, $Y_{TR3}^{(2)}$ and $Y_{TR4}^{(2)}$ of dimensions (6x6),(6x3),(3x6) and (3x3) respectively
6	Three-phase line L2	16-18	19-21	$Y_{TL}^{(2)}$	With submatrices Y_{L2}^3, $-Y_{L2}^3$, $-Y_{L2}^3$ and Y_{L2}^3 all of dimension (3x3)

The partial network at this stage is described by the following nodal admittance matrix :

$$Y = \begin{array}{c|c|} & 1 \quad \cdots \quad 3 \\ \hline 1 & \\ \vdots & Y_G^3 \\ 3 & \\ \hline \end{array} \tag{3.5}$$

Step 2 : The next element is a wye/star transformer T_1 described by the following nodal admittance matrix between busgroups (1-3) and (4-9) as given by eqn. (2.15) as

$$Y_{TR}^{(1)} = \begin{array}{c|c|c|} & 1 \ \cdots \ 3 & 4 \ \cdots \ 9 \\ \hline 1 \\ \vdots & Y_{TR1}^{(1)} & Y_{TR2}^{(1)} \\ 3 \\ \hline 4 \\ \vdots & Y_{TR3}^{(1)} & Y_{TR4}^{(1)} \\ 9 \\ \hline \end{array} \tag{3.6}$$

After adding this element submatrix (3.6) appropriately to (3.5), the partial network at this stage is described by the following nodal admittance matrix

$$
Y = \begin{array}{c|c|c|}
 & 1 \quad \cdots \quad 3 & 4 \quad \cdots \quad 9 \\
\hline
\begin{array}{c}1\\ \vdots\\ 3\end{array} & Y_G^3 + Y_{TR1}^{(1)} & Y_{TR2}^{(1)} \\
\hline
\begin{array}{c}4\\ \vdots\\ 9\end{array} & Y_{TR3}^{(1)} & Y_{TR4}^{(1)} \\
\hline
\end{array}
\qquad (3.7)
$$

Step 3 : The next element to be added is a three-phase line L3 connected between bus groups (1-3) and (19-21) described by the following nodal admittance matrix

$$
Y_{TL}^{(3)} = \begin{array}{c|c|c|}
 & 1 \quad \cdots \quad 3 & 19 \quad \cdots \quad 21 \\
\hline
\begin{array}{c}1\\ \vdots\\ 3\end{array} & Y_{L3}^3 & -Y_{L3}^3 \\
\hline
\begin{array}{c}19\\ \vdots\\ 21\end{array} & -Y_{L3}^3 & Y_{L3}^3 \\
\hline
\end{array}
\qquad (3.8)
$$

After adding this element submatrix (3.8) appropriately to (3.7), the partial network at this stage is described by the following nodal admittance matrix.

$$
Y = \begin{array}{c|c|c|c|}
 & 1 \cdots 3 & 4 \cdots 9 & 19 \cdots 21 \\
\hline
1 \cdots 3 & Y_G^3 + Y_{TR1}^{(1)} + Y_{L3}^3 & Y_{TR2}^{(1)} & -Y_{L3}^3 \\
\hline
4 \cdots 9 & Y_{TR3}^{(1)} & Y_{TR4}^{(1)} & \\
\hline
19 \cdots 21 & -Y_{L3}^3 & & Y_{L3}^3 \\
\hline
\end{array} \qquad (3.9)
$$

Step 4 : The next element is a six-phase line L1 described by the following nodal admittance matrix between bus groups (4-9) and (10-15) given by eqn. (2.18) as,

$$
Y_{TL}^{(1)} = \begin{array}{c|c|c|}
 & 4 \cdots 9 & 10 \cdots 15 \\
\hline
4 \cdots 9 & Y_{L1}^6 & -Y_{L1}^6 \\
\hline
10 \cdots 15 & -Y_{L1}^6 & Y_{L1}^6 \\
\hline
\end{array} \qquad (3.10)
$$

After adding this element submatrix (3.10) appropriately to (3.9), the partial network matrix at this stage is described by the following nodal admittance matrix;

$$
Y = \begin{array}{c} 1 \\ \vdots \\ 3 \\ 4 \\ \vdots \\ 9 \\ 10 \\ \vdots \\ 15 \\ 19 \\ \vdots \\ 21 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 \; \ldots \; 3 & 4 \; \ldots \; 9 & 10 \; \ldots \; 15 & 19 \; \ldots \; 21 \end{array} \\
\left[\begin{array}{c|c|c|c}
Y_G^3+Y_{TR1}^{(1)}+Y_{L3}^3 & Y_{TR2}^{(1)} & & -Y_{L3}^3 \\ \hline
Y_{TR3}^{(1)} & Y_{TR4}^{(1)}+Y_{L1}^6 & -Y_{L1}^6 & \\ \hline
 & -Y_{L1}^6 & Y_{L1}^6 & \\ \hline
-Y_{L3}^3 & & & Y_{L3}^3
\end{array}\right]
\end{array}
\qquad (3.11)
$$

Step 5 : The next element is a star/wye transformer T_2 described by the following nodal-admittance matrix derived from eqn. (2.15) as

$$
Y_{TR}^{(2)} = \begin{array}{c} 10 \\ \vdots \\ 15 \\ 16 \\ \vdots \\ 18 \end{array}
\begin{array}{c}
\begin{array}{cc} 10 \; \ldots \; 15 & 16 \; \ldots \; 18 \end{array} \\
\left[\begin{array}{c|c}
Y_{TR1}^{(2)} & Y_{TR2}^{(2)} \\ \hline
Y_{TR3}^{(2)} & Y_{TR4}^{(2)}
\end{array}\right]
\end{array}
\qquad (3.12)
$$

After adding this element submatrix (3.12) appropriately to (3.11), the partial network at this stage is described by the following nodal admittance matrix,

$$
Y = \begin{array}{c|c|c|c|c|c|}
 & 1 \;\cdots\; 3 & 4 \;\cdots\; 9 & 10 \;\cdots\; 15 & 16 \;\cdots\; 18 & 19 \;\cdots\; 21 \\
\hline
1 \cdots 3 & Y_G^3+Y_{TR1}^{(1)}+Y_{L3}^3 & Y_{TR2}^{(1)} & & & -Y_{L3}^3 \\
\hline
4 \cdots 9 & Y_{TR3}^{(1)} & Y_{TR4}^{(1)}+Y_{L1}^6 & -Y_{L1}^6 & & \\
\hline
10 \cdots 15 & & -Y_{L1}^6 & Y_{L1}^6+Y_{TR1}^{(2)} & Y_{TR2}^{(2)} & \\
\hline
16 \cdots 18 & & & Y_{TR3}^{(2)} & Y_{TR4}^{(2)} & \\
\hline
19 \cdots 21 & -Y_{L3}^3 & & & & Y_{L3}^3 \\
\hline
\end{array}
\tag{3.13}
$$

Step 6 : The last element is a three-phase line L2 described by the following nodal admittance matrix between bus groups (16-18) and (19-21) as

$$
Y_{TL}^{(2)} = \begin{array}{c|c|c|}
 & 16 \;\cdots\; 18 & 19 \;\cdots\; 21 \\
\hline
16 \cdots 18 & Y_{L2}^3 & -Y_{L2}^3 \\
\hline
19 \cdots 21 & -Y_{L2}^3 & Y_{L2}^3 \\
\hline
\end{array}
\tag{3.14}
$$

After adding this last element submatrix (3.14) appropriately to (3.13), the final network system nodal admittance matrix is given by

$$
\begin{array}{c|c|c|c|c|c|}
 & 1 \;\dots\; 3 & 4 \;\dots\; 9 & 10 \;\dots\; 15 & 16 \;\dots\; 18 & 19 \;\dots\; 21 \\
\hline
1 \;\cdot\; 3 & Y_G^3+Y_{TR1}^{(1)}+Y_{L3}^3 & Y_{TR2}^{(1)} & & & -Y_{L3}^3 \\
\hline
4 \;\cdot\; 9 & Y_{TR3}^{(4)} & Y_{TR4}^{(1)}+Y_{L1}^6 & -Y_{L1}^6 & & \\
\hline
10 \;\cdot\; 15 & & -Y_{L1}^6 & Y_{L1}^6+Y_{TR1}^{(2)} & Y_{TR2}^{(2)} & \\
\hline
16 \;\cdot\; 18 & & & Y_{TR3}^{(2)} & Y_{TR4}^{(2)}+Y_{L2}^3 & -Y_{L2}^3 \\
\hline
19 \;\cdot\; 21 & -Y_{L3}^2 & & & -Y_{L2}^3 & Y_{L3}^3+Y_{L2}^3 \\
\hline
\end{array}
$$

(3.15)

Substituting the numerical values of different submatrices in (3.15), a (21x21) order nodal admittance matrix is obtained. The sequence in which the elements are added is arbitrary since after the admittance submatrix of the element (being added), has been calculated, it is transferred to the corresponding rows and columns of partial system network admittance matrix. The program to generate the system nodal admittance matrix is written in modular fashion incorporating several features of representations of multi-phase system and the flexibility to make network changes

required during the course of system studies. A simplified flow chart in Fig. 3.4 indicates the logical development of the program.

3.3 LOAD FLOW STUDIES

Load flow is a basic analytical tool required for planning and operation of power systems. Although the load flow methods for three-phase systems are well established [28-35], only a balanced single-phase load flow study [2] has been carried out primarily to establish the increased power capabilities of six-phase transmission line. However, if an integrated three-phase and six-phase system is to be analysed, three-phase techniques and programs need modifications and extensions to include the suitable representations of six-phase (multi-phase) elements. In this section, the procedure for load flow studies are developed for a completely six-phase and also for a composite three-phase and six-phase system for balanced as well as unbalanced conditions.

3.3.1 Balanced Single Phase Load Flow Analysis

The representations of a six-phase and a composite three-phase and six-phase system on equivalent single phase basis has been discussed in Section 3.2.1. Employing these representations, the load flow analysis is carried out as usual.

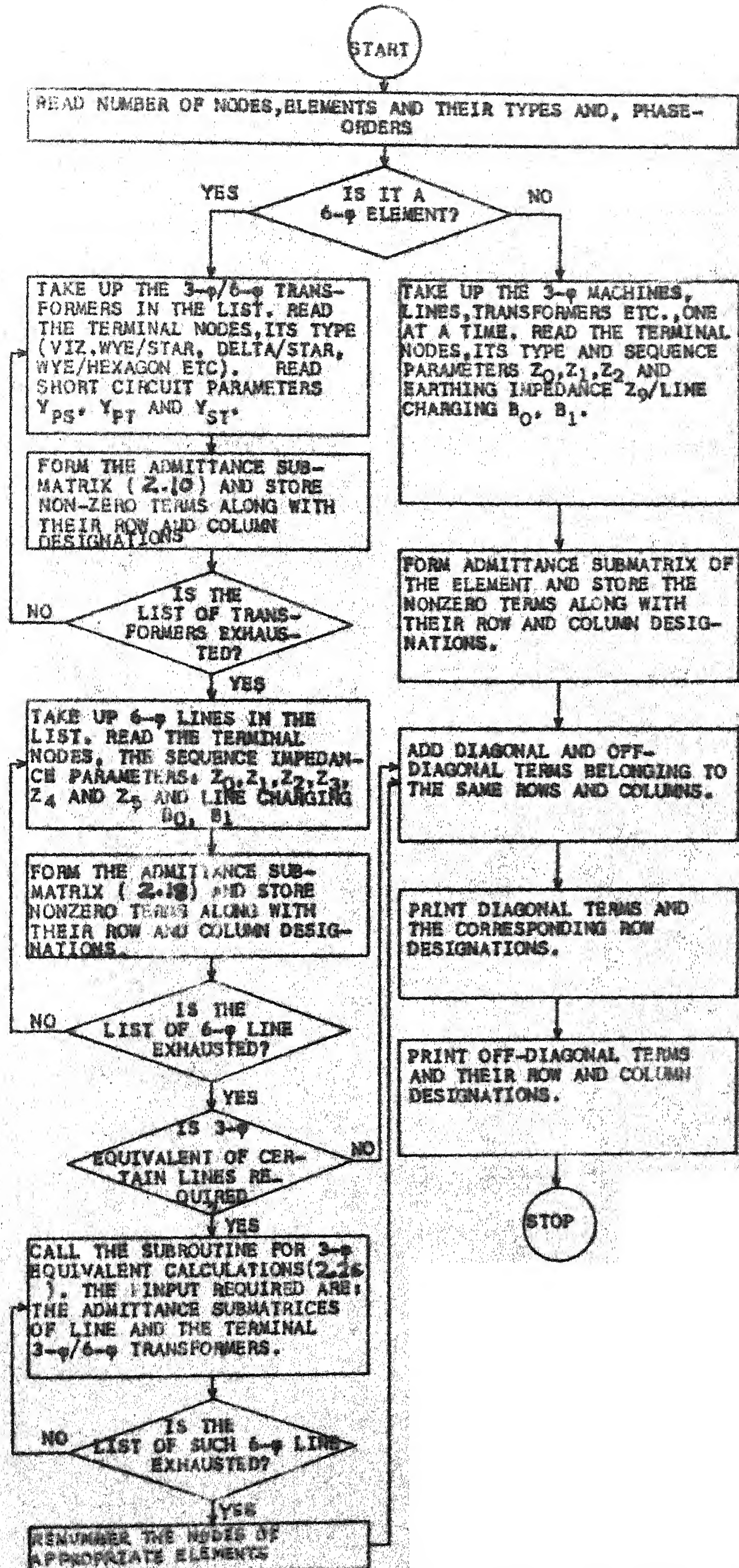


FIG. 3.4 A SIMPLIFIED FLOW CHART FOR MULTI-PHASE BUS ADMITTANCE MATRIX ASSEMBLY PROGRAM

3.3.2 Phase Coordinate Load Flow Analysis

With the increasing complexities of present day power systems involving parallel untransposed multi-circuit lines and the resulting circulating currents, parameter and load unbalances, simultaneous occurrences of faults etc. and th. need to investigate them precisely, the interest has grown in the use of methods using phase coordinate representations. The transformation methods [23] have had computational advantages in the past but with the availability of large and fast digital computers, they no longer appear to be necessary. The phase coordinate method employs primary or untransformed reference frame variables. Since there is one to one correspondence between the elements of the nodal admittance matrix and the system network, the physical identities of different elements areretained. The load flow analysis employing phase coordinate representations has been discussed in several papers for conventional three-phase system [28-34] which is extended here to six-phase and a composite three-phase and six-phase system. The approach [27] involves the following main steps;

i) Representation of various elements of the system in phase frame of reference

ii) Assembly of system multi-phase nodal admittance matrix

iii) Nodal admittance matrix modification to simulate changes in system network configuration

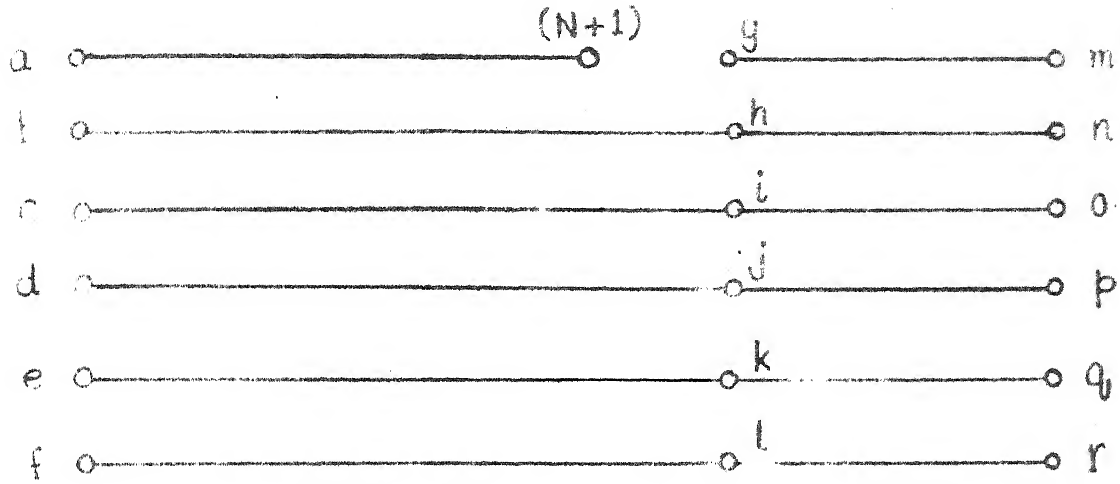


FIG. 3.5 ILLUSTRATION OF ONE CONDUCTOR OPENING

$$Y_{(N+1)\,x} = Y_{g\,x}$$

$$Y_{gx} = 0 \; ; \quad x = a,b, \ldots f \tag{3.16}$$

$$Y_{gx} = -Y_{mx}$$

$$Y_{(N+1)x} = -Y_{ax} \; ; \quad x = h,i, \ldots,\ell \tag{3.17}$$

$$Y_{(N+1)\,g} = 0$$

$$Y_{(N+1)\,(N+1)} = \quad Y_{(N+1)x} : x = a,b \ldots f,\; h,i, \ldots l$$

$$\tag{3.18}$$

$$Y_{gg} = \quad Y_{gx} : \quad x = m,n \ldots r,h,i, \ldots l \tag{3.19}$$

A phase opening creates two additional nodes. The base-case matrix may be modified in a similar fashion. The simulation of more than one conductor or phase opening is carried out merely by repeating the procedure as discussed.

Other changes in the network

The addition of a line or element is incorporated by calculating its admittance submatrix and integrating it to the system base case matrix appropriately. Similarly, the removal of an element which is not mutually coupled to any other element is simulated by removing its admittance submatrix from the base case matrix. The change of an impedance parameters such as off nominal tappings of

transformers, etc., change of transformer connections, change in type of load representations etc. is carried out conveniently by replacing the earlier element admittance sub matrix by the new ones in the system base case matrix. The development of multi-phase bus admittance matrix in modular fashion facilitates these changes without affecting the remaining portion of the program.

(iv) Problem formulation and solution

A phase coordinate load flow is handled much like a single phase load flow, where each single phase voltage, current and power become a multi-phase (i.e. three-phase or six-phase as the case may be) vector and each single phase admittance is replaced with multi-phase admittance matrix of appropriate dimension. The solution steps may be summarised as;

i) Each phase of a multi-phase bus including the neutrals of machines, transformers etc. is assigned a node number.

ii) Each element admittance submatrix is generated separately and then combined to form the multi-phase bus admittance matrix (3.1)

iii) The load flow data prepared in accordance with the phase numbering in (i) is supplied.

iv) Eqns. (3.1) are solved iteratively by any one of the standard numerical techniques for the solution of simultaneous algebraic equations. However, the Gauss

Siedel technique is employed in this work for the solution of power flow equations.

v) The slack bus powers, line flows and line losses are calculated as usual.

The phase coordinate technique of load flow is a very convenient and powerful tool to analyse complex network situations. The method is applicable to balanced as well as unbalanced conditions. However, the method may be made attractive for simulating large scale systems with the judicious trade off between the degree of representations needed, interest of investigation, accuracy, memory and computational requirements. In view of these considerations, the following schemes may be considered.

Scheme I (Three-phase Load flow)

If the interest of investigation and the unbalances lie in three-phase part of the network, the six-phase elements may usefully be replaced by their three-phase equivalents and the analysis of the entire system may be carried out on equivalent three-phase basis.

Scheme II (Six-phase load flow)

When the interest of investigation is centred in six-phase part of the network, the three-phase elements may be replaced by their six-phase equivalents. This scheme is generally not to be preferred because of the prohibitive memory requirement.

Scheme III (Mixed three-phase and six-phase load flow)

The scheme retains the physical identities of three-phase and six-phase elements and may be employed to investigate the problem of unbalances existing in either three-phase or six-phase or both parts of the network.

The various load flow methods and schemes, as discussed, have been applied on several sample networks to check the validity of the programs developed for the purpose. However, by way of demonstration, only a few results of balanced and unbalanced load flow studies are consolidated here.

Numerical Examples

Normal (Balanced) Load Flow Analysis

An equivalent single-phase, three-phase, and mixed three-phase and six-phase load flow studies were carried out for the network of Fig. 3.6. The necessary data are given in Table 3.3. For taking a comparative view of the results obtained in the three alternative procedures, the voltage magnitudes and phase angles (only for the phase A buses in phase coordinate methods) are recorded in Table 3.4. The results obtained by all the three methods are in close agreement.

Table 3.3

Circuit characteristics with 83.333 MVA Base/Phase and Rated KV

Line No.	Type	R (p.u.)	X (p.u.)	Y_{sh} (p.u.)
L1	2	0.0388902	0.231700	0.0459988
	3	0.0259268	0.162781	0.0654138
L2	1	0.0777804	0.517880	0.0205180
L3	1	0.0777804	0.517880	0.0205180

where 1,2,3 indicate single circuit three-phase, double circuit three-phase and six-phase line respectively.

Table 3.4

Summary of results of normal (balanced) load flow analysis

Bus No.	ESP Load flow		Phase A Bus No.	ETP load flow		Phase A Bus No.	MTPSP Load Flow	
	$\lvert V\rvert$ (p.u.)	θ (deg)		$\lvert V\rvert$ (p.u.)	θ (deg)		$\lvert V\rvert$ (p.u.)	θ (deg)
1	1.041	-5.415	1	1.041	-5.413	1	1.041	-5.413
2	0.999	-13.206	4	0.999	-13.189	16	0.999	-13.203
3	0.966	-16.910	7	0.966	-16.892	19	0.966	-16.901
4	1.070	0.0	10	1.07	0.0	22	1.07	0.0

where ESP - equivalent single phase; ETP - equivalent three-phase and MTPSP - mixed three-phase and six-phase respectively.

Unbalanced Load Flow

Table 3.5 presents the results of phase coordinate load flow for the network of Fig. 3.6 (redrawn in Fig. 3.7 for conceptual clarity) employing Scheme III for unbalanced phase loadings at buses 16,17 and 18 (as 120 percent, 100 percent, 80 percent of their base case values) and off nominal tap setting of transformer T_1 on secondary side at -0.025.

Results and Discussion

The results of normal (balanced) load flows presented in Table 3.4 show a close agreement between the three alternative methods. The three methods however, present the system operating conditions in varying degree of details with the mixed three-phase and six-phase load flow yielding complete information about the system. As such, the computational times for the solution may vary significantly in different procedures depending upon the accuracy required and system size. To obtain an estimate of the computational time required in the three alternative methods, the system of Fig. 3.9 was worked out. The computational times for a convergence of 0.000002 were found to be 1.24 sec, 2 minutes 5 sec., and 5 minutes 47 sec. respectively.

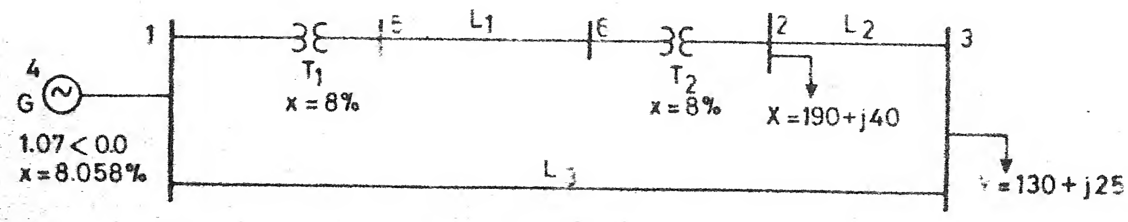


FIG. 3.6 SINGLE LINE DIAGRAM OF A SAMPLE SYSTEM WITH LOADS MARKED IN MVA

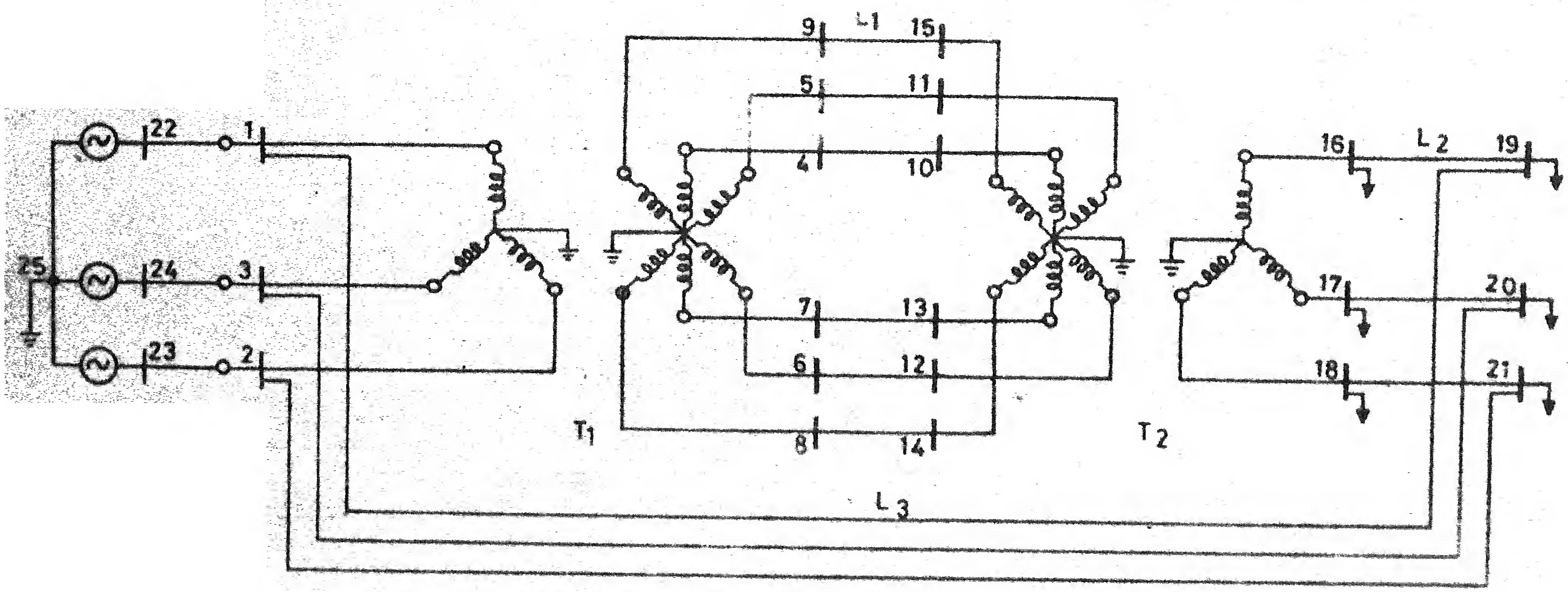


FIG. 3.7 SCHEMATIC DIAGRAM OF NETWORK OF FIG. 3.6 SHOWING BUS NUMBERING SEQUENCE FOR PHASE COORDINATE LOAD FLOW AND FAULT ANALYSIS (SCHEME III)

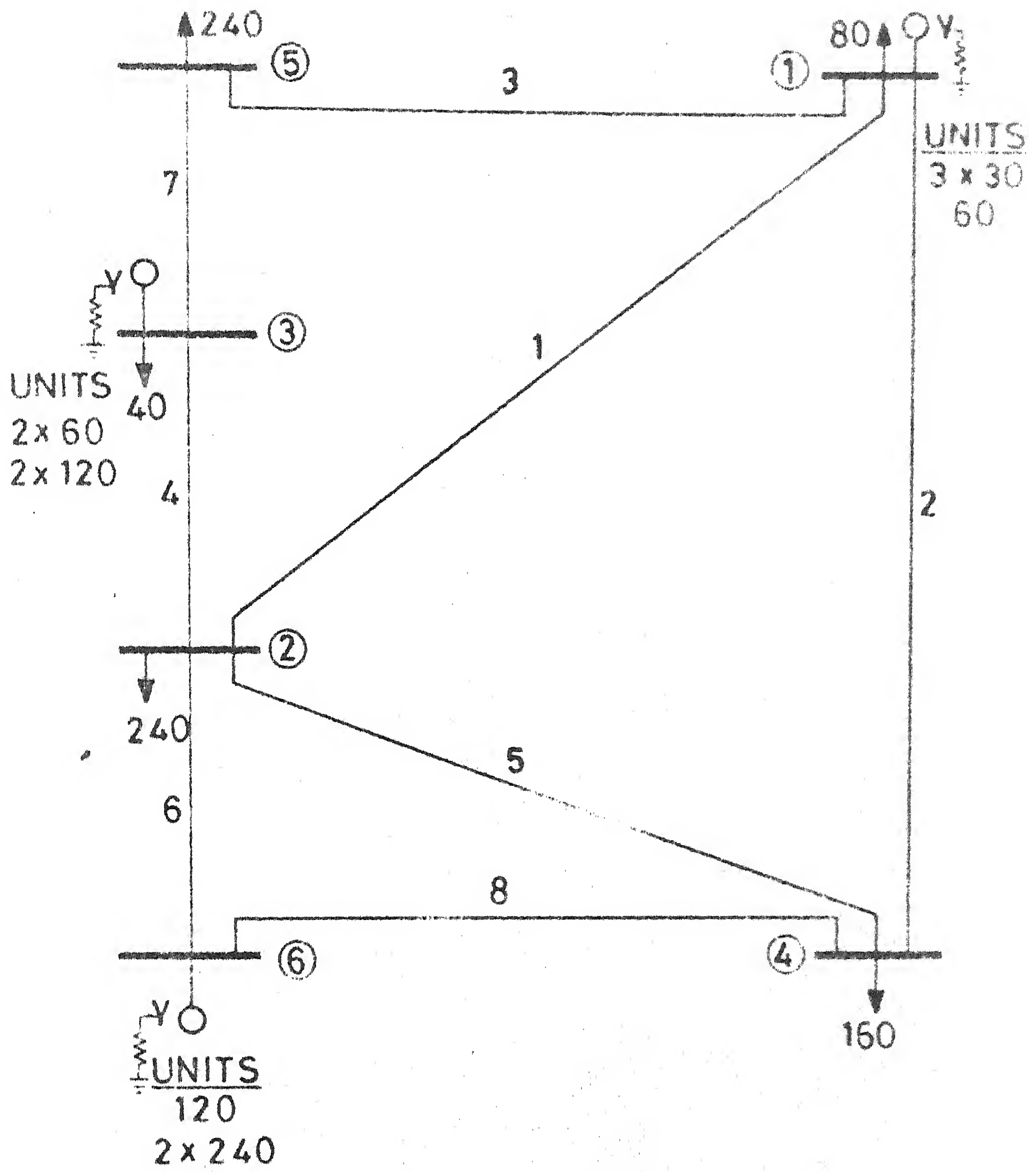


FIG. 3.8 SINGLE LINE DIAGRAM OF A SAMPLE SYSTEM WITH LOADS AND GENERATION IN MW

Table 3.5

Voltage solution for phase coordinate load flows for the network of Fig. 3.7

Bus No.	Unbalanced Case |V| (p.u.)	Unbalanced Case θ (deg)	Balanced Case |V| (p.u.)	Balanced Case θ (deg)
1	1.035	-6.01	1.041	-5.41
2	1.036	-125.37	1.041	-125.48
3	1.048	115.09	1.041	114.66
4	1.000	-8.24	1.034	-7.32
5	1.018	-66.46	1.034	-67.25
6	1.004	-127.29	1.034	-127.39
7	1.000	171.75	1.034	172.67
8	1.018	113.53	1.034	112.74
9	1.004	52.70	1.034	52.60
10	0.966	-13.01	1.007	-11.18
11	0.998	-69.79	1.007	-71.10
12	0.978	-131.38	1.007	-131.25
13	0.966	166.98	1.007	168.81
14	0.998	110.21	1.007	108.89
15	0.978	48.61	1.007	48.74
16	0.956	-15.54	0.999	-13.20
17	0.970	-133.52	0.999	-133.27
18	0.991	108.49	0.999	106.86
19	0.933	-18.63	0.966	-16.90
20	0.947	-137.60	0.966	-136.98
21	0.971	104.43	0.966	103.15
22	1.070	0.0	1.070	0.0
23	1.070	-120.0	1.070	-120.0
24	1.070	120.0	1.070	120.0
25	0.0	0.0	0.0	0.0

From the results presented in Table 3.5, it can be seen that the system, in balanced conditions, yields a balanced performance with respect to both three-phase and six-phase parts. In unbalanced conditions, the combined effect of load unbalancing at buses 16-18 and off nominal tap setting of transformer T_1 (as -0.025 on secondary side) is obtained, and shown along with the base case solution for a comparative examination.

3.4 CASE STUDIES

In order to study the performance of six-phase (multi-phase) systems with particular reference to six-phase transmission, from load flow considerations, certain specific studies were carried out on some sample networks. Few important case studies and their results are presented and discussed.

1. Load Flow Performance of Six-Phase (Multi-phase) System

With a view to obtain relative performance of six-phase systems as compared to conventional three-phase systems, the sample system of Fig. 3.6 is modified and classified as :

A A completely three-phase system : with the system details given in Table 3.3 and without transformers T_1 and T_2.

B A composite three-phase and six-phase system : with the following modifications in

(i) Line L1 is converted to a six-phase line implying phase-ground voltage in six-phase circuit increased to $\sqrt{3}$ times that of the original three phase line.

(ii) T_1 and T_2 are three-phase/six-phase, wye/star transformers.

C A completely six-phase system : with the following modifications in B

(i) Generator G is a six-phase generator with the same per phase power rating and impedance as that of three phase

(ii) Lines L2 and L3 are replaced by six-phase lines with same per phase values as in three-phase

(iii) T_1 and T_2 are six-phase, star/star transformers.

The results of load flow studies for fixed and increased phase loadings for system configuration A,B and C are presented vide Tables 3.6 and 3.7.

Remark :

A careful examination of Tables 3.6 and 3.7 reveals that the performance of high phase order systems is better in terms of voltage magnitudes and phase angles than their lower phase order counterparts even for higher system loadings.

Table 3.6

Load flow solution for system configurations A,B and C for fixed system loadings

Bus No.	Type of Bus	System loadings X = 190 + j40 and Y = 130 + j25					
		System A		System B		System C	
		|V| (p.u.)	θ (deg)	|V| (p.u.)	θ (deg)	|V| (p.u.)	θ (deg)
1	P,Q	1.027	-5.600	1.041	-5.418	1.062	-2.635
2	P,Q	0.938	-16.491	0.999	-13.203	1.031	-9.133
3	P,Q	0.922	-19.162	0.966	-16.901	1.026	-9.368
4	Slack	1.070	0.0	1.070	0.0	1.070	0.0

Table 3.7

Load flow solutions for system configurations B and C for increased loadings

Bus No.	Type of Bus	System loadings X = 330 + j70; Y = 130 + j25				System loadings X = 380 + j80; Y = 260+j50	
		System B		System C		System C	
		|V| (p.u.)	θ (deg)	|V| (p.u.)	θ (deg)	|V| (p.u.)	θ (deg)
1	P,Q	0.998	-8.210	1.048	-3.868	1.020	-5.615
2	P,Q	0.906	-22.115	0.983	-14.544	0.912	-20.880
3	P,Q	0.886	-23.840	0.991	-12.809	0.900	-21.436
4	Slack	1.070	0.0	1.070	0.0	1.070	0.0

2. Impact of converting certain double circuit three-phase lines to six-phase lines

As the six-phase (multi-phase) transmission is being considered as an alternative to double circuit three phase transmission system, its viability needs a careful examination. With this in view, the sample system of Fig. 3.8, where lines 6-8 are originally operating as double circuit three-phase lines, is considered. The lines 6-8 are converted to six-phase lines, stressed to the same phase-phase (adjacent) voltages. The conversion of the three-phase double circuit line to six-phase line would mean an increase in phase-ground voltage by a factor of $\sqrt{3}$. This in principle is equivalent to uprating the existing line with the voltage increased by $\sqrt{3}$. The adequacy of the stipulated changes made are studied carrying out the load flow analyses. The possibility of meeting more loads with six-phase conversion of lines 6-8 is also studied. The circuit characteristics, the necessary load flow data and the results are given in Tables 3.8 - 3.12.

From an examination of load flow study results (Table 3.11 and Table 3.12) the following observations are made.

i) The voltage magnitudes and phase angles of all nodes improve substantially (case II, Table 3.11) as the lines 6-8 are converted to six-phase lines, maintaining same

line-line (adjacent) voltages and conductor configurations. Even with increased loadings of 1.66 times the original system loadings, the benefits of improved voltage magnitude and phase angles are retained to an appreciable extent as evident from the values of (P,Q) nodes 2,4 and 5. This means that the better voltage regulation (or MVAR control) is obtained by replacing double circuit three phase lines by six-phase lines.

Table 3.8

Circuit characteristics on 100 MVA base

Line No.	Type	R (p.u.)	X (p.u.)	Normal capacity (MW)
1	1	0.10	0.40	100
2	1	0.15	0.60	80
3	1	0.05	0.20	100
4	1	0.05	0.20	100
5	1	0.10	0.40	100
6	2 (3)	0.01875 (0.00625)	0.075 (0.04357)	400 (692)
7	2 (3)	0.025 (0.00833)	0.10 (0.07143)	200 (346)
8	2 (3)	0.0375 (0.0125)	0.15 (0.08715)	200 (346)

where entries in column 2 have the following meanings

1 - single circuit three phase line, 2 - double circuit three phase line, 3 - six-phase line, ( ) - the quantities in bracket refer to six-phase lines)

Table 3.9

A.C. load flow data for the original and modified network

Bus No.	Type	Generation (MVA)	Load (MVA)	Voltage magnitude (p.u.)	Phase angle (degrees)
1	P, V	51 + j0.0	80 + j0.0	1.02	0.0
2	P,Q	0.0 + j0.0	240 + j0.0	1.04	0.0
3	P, V	168 + j0.0	39.7 + j0.0	1.04	0.0
4	P,Q	0.0 + j0.0	160 + j40.0	1.04	0.0
5	P,Q	0.0 + j0.0	240 + j0.0	1.04	0.0
6	Slack	0.0 + j0.0	0.0 + j0.0	1.04	0.0

Table 3.10

A.C. load flow data for increase system loading

Bus No.	Type	Generation (MVA)	Load (MVA)	Voltage magnitude (p.u.)	Phase angle (degrees)
1	P, V	51 + j0.0	80 + j0.0	1.02	0.0
2	P,Q	0.0 + j0.0	530 + j0.0	1.04	0.0
3	P, V	240 + j0.0	111.70 + j0.0	1.04	0.0
4	P,Q	0.0 + j0.0	200 + j40	1.04	0.0
5	P,Q	0.0 + j0.0	240 + j0.0	1.04	0.0
6	Slack	0.0 + j0.0	0.0 + j0.0	1.04	0.0

Table 3.11

Voltage magnitudes and phase angles

Bus No.	Case I Original network System load=759.70MW		Case II 6,7,8 are 6φ-lines System load=759.70 MW		Case III 6,7,8 are 6-φ lines System load=1261.70 MW	
	$\lvert V \rvert$ (p.u.)	θ (deg)	$\lvert V \rvert$ (p.u.)	θ (deg)	$\lvert V \rvert$ (p.u.)	θ (deg)
1	1.02	-31.42	1.02	-19.97	1.02	-28.88
2	0.96	-16.87	1.02	-8.75	0.98	-16.55
3	1.04	-26.59	1.04	-17.45	1.04	-26.28
4	0.90	-17.27	0.98	-8.98	0.94	-16.81
5	0.98	-37.29	1.01	-25.08	1.01	-33.93
6	1.04	0.0	1.04	0.0	1.04	0.0

Table 3.12

Line flows and transmission efficiencies

Line No.	From bus	To bus	Case I Original network System load=759.70MW			Case II 6,7 and 8 are 6-φ lines System load=759.70MW			Case III 6,7 and 8 are 6-φ lines System load=1261.70MW		
	i	j	P_{ij} (MW)	Q_{ij} (MVAR)	η (%)	P_{ij} (MW)	Q_{ij} (MVAR)	η (%)	P_{ij} (MW)	Q_{ij} (MVAR)	η (%)
1	1	2	-52.96	35.31	93.08	-46.08	17.45	95.16	-46.15	28.17	94.26
2	1	4	-29.64	31.76	91.59	-27.83	16.20	94.88	-27.26	24.26	93.38
3	1	5	53.60	10.31	97.33	44.91	-3.68	97.84	44.42	-3.62	97.83
4	2	3	72.80	-47.46	94.42	73.84	-24.47	95.96	75.29	-42.17	94.82
5	2	4	4.84	13.30	95.66	2.91	7.54	97.93	3.23	8.76	96.90
6	2	6	-374.50	53.90	92.84	-365.17	25.04	97.82	-657.48	50.34	98.85
7	3	5	197.04	32.27	95.32	199.20	34.59	98.41	199.69	34.56	98.41
8	4	6	-187.73	-6.68	92.06	-168.47	-22.48	97.61	-326.05	-15.02	95.56

(ii) The real and reactive power loadings of most of the lines are reduced. The benefit is relatively more quantitative in reactive flows.

(iii) The line efficiencies are improved meaning reduced losses in the system. Lines 6-8 in particular show marked improvements in efficiencies in all the cases.

(iv) The system can transfer more load. This was investigated by increasing the system load from 759.7 MW to 1261.7 MW (1.66 times the original load). The data and results are given in Tables 3.10 - 3.12 respectively, assuming that the additional generating capacities required may be added at nodes 3 and 6. As evident from the results, the increased load is supplied without overloading any of the lines, while still maintaining appreciable benefits in voltage regulation and line efficiencies.

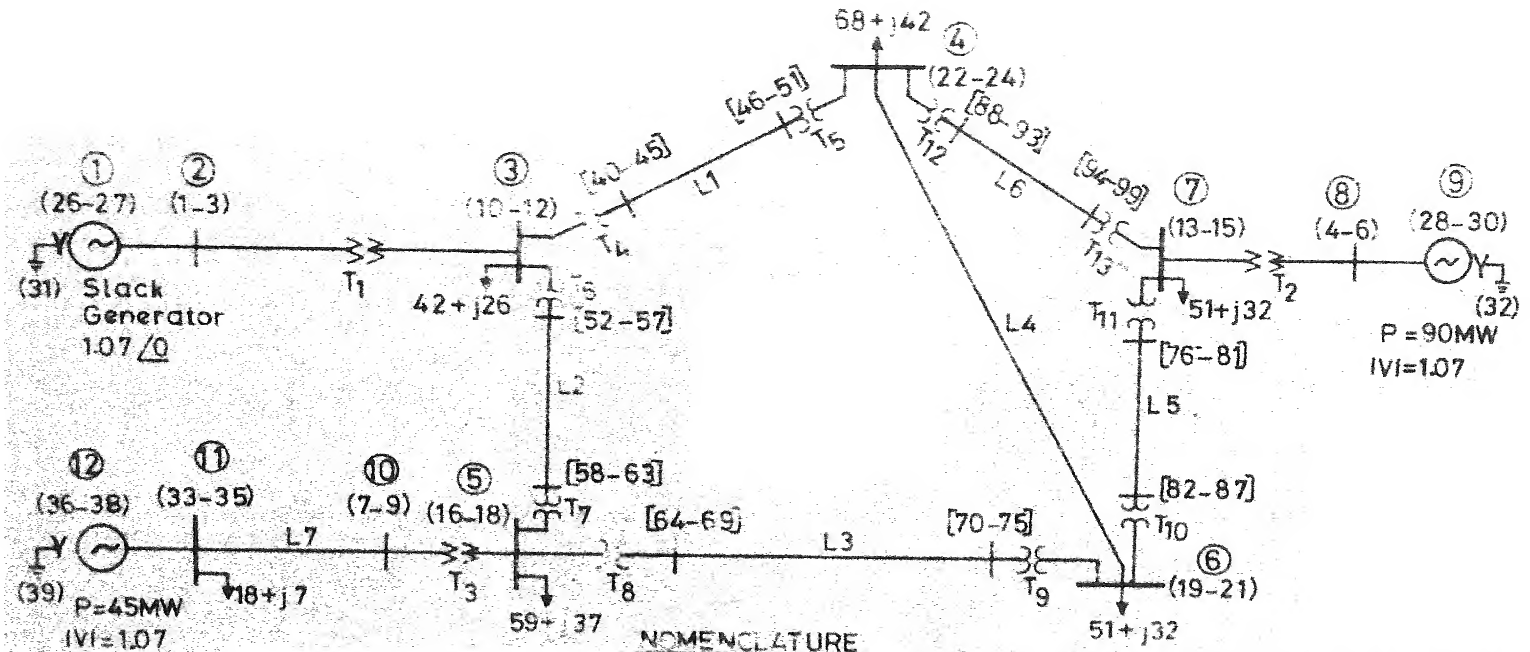
The above observations in performance may be explained by examining the values of parameters (in p.u.) for double circuit three-phase lines and six-phase lines in Table 3.10. As the p.u. series parameters of six-phase lines are significantly lower than those of double circuit three-phase lines, the voltage drop in the line becomes less causing voltage of the nodes to improve both in magnitudes and phase angles. The shunt parameters (i.e. line charging admittance) increase in

the same ratio which further helps to improve voltage and decrease reactive power requirements. (However, the shunt parameters are neglected in this example and therefore, they do not enter into the calculations). For the same system load (Table 3.12, Case II), the current and power flows in most of the lines are lower in values as compared to Case I (Table 3.12). The lower values of resistance and real power flows of six-phase lines tend to improve the transmission efficiencies. The distribution of these benefits is governed mainly by the topology of the network, location of loads and generating centres. The higher power transfer capability of six-phase line is due to the fact that phase - neutral voltage in six-phase system is $\sqrt{3}$ times than that in the case of three-phase double circuit line. Thus, ideally, a six-phase line can transmit 1.732 times power than double circuit three phase line. However, the leakage impedance of the interfacing transformers modifies the figure to a lower value. Again because of the lower series parameters and higher shunt parameters even with more power flows for higher system loads (Case III, Table 3.12), the benefits in voltage regulation and line efficiencies are maintained to an appreciable extent. In fact, as the number of six-phase lines added in the existing network increases, the improvements in system performance also increase proportionately.

4. Simulation of Large Scale System

The phase coordinate load flow schemes I, II and III presented in previous sections need further considerations while applying them to large systems. As discussed earlier, if the interest of investigation and unbalances lie in three phase part of the network, the entire system may be analysed on equivalent three-phase basis (Scheme I). Scheme II becomes restrictive for large systems because of prohibitive memory requirements. Scheme III is a good compromise between Schemes I and II and retains the physical identities of different elements. However, a further saving in memory may be obtained by representing only those six-phase elements where the unbalances exist and/or the main interest of investigation lies, and the rest of the elements represented by their three-phase equivalents. It is to be noted that sparsity and its effective utilization become much more important than single phase analysis, especially, when large size systems are solved. The other problems that need careful consideration are the computational time, accuracy and availability of effective starting solutions. Some of these aspects are investigated on a sample system depicted in Fig. 3.9 and data supplied in Table 3.13.

In carrying out the various cases of load flow employing Schemes I and III, the following starting solutions were employed.



NOMENCLATURE

○	Bus numbering sequence for equivalent single phase load flow
( )	Bus numbering sequence for equivalent three phase load flow
[ ]	Six phase bus bar groups
( ) & [ ]	Bus numbering sequence for phase coordinate load flow

FIG. 3.9 A SAMPLE SYSTEM SHOWING BUS NUMBERING SEQUENCE FOR EQUIVALENT SINGLE AND THREE-PHASE, AND MIXED THREE-PHASE AND SIX-PHASE REPRESENTATION FOR LOAD FLOW AND FAULT ANALYSIS

Table 3.13

Circuit characteristics for sample system of Fig.3.9 on 100 MVA base/phase and rated kV

From nodes	To nodes	±ve sequence			Zero sequence		
		R_1	X_1	B_1	R_o	X_o	B_o
1) Generators							
(1-3)	(25-27)	0.0	0.0967	-	0.0	0.0467	-
(4-6)	(28-30)	0.0	0.17	-	0.0	0.085	-
(33-35)	(36-38)	0.0	0.17	-	0.0	0.085	-
2) 3-φ Lines							
(19-21)	(22-24)	0.0055	0.0248	0.0041	0.0171	0.0729	0.0026
(7-9)	(33-35)	0.0073	0.0330	0.0054	0.0228	0.0972	0.0034
3) 6-φ Lines							
[40-45]	[46-51]	0.0097	0.0464	0.0156	0.0572	0.3605	0.0038
[52-57]	[58-63]	0.0073	0.0344	0.0114	0.0431	0.2677	0.0028
[64-69]	[70-75]	0.0061	0.0290	0.0097	0.0357	0.2256	0.0024
[76-81]	[82-87]	0.0024	0.0116	0.0038	0.0141	0.0901	0.0010
[88-93]	[94-99]	0.0049	0.0232	0.0077	0.0286	0.1802	0.0019

4) 3-φ Transformers

From nodes	To nodes	Type of connection	X
(1-3)	(10-12)	delta/wye	0.0533
(4-6)	(13-15)	delta/wye	0.12
(7-9)	(16-18)	delta/wye	0.16

5) 3-φ/6-φ Transformers

All transformers T4-T13 are wye/star, solidly earthed with $y_{PS} = y_{PT} = y_{ST} = -j12.5$.

a. Flat starting solution : Assuming voltage magnitude equal to 1. p.u. and phase angle 0 (with three-phase and six-phase balance) for all buses except the slack bus and other buses with specified values.

b. Equivalent single phase/three phase solution : assuming three and six-phase balance.

c. Normal (base case) phase coordinate solution

The results of the investigations i.e. number of iterations, computational time etc. are recorded for each case in Table 3.14 for a quantitative comparison. The method used in each case is the Gauss Siedel iterative technique with acceleration factor of 1.2 and the convergence requirements of 0.0002. The computations were carried out on DEC-1090 system at I.I.T. Kanpur, India.

Since the sample system (Fig. 3.6) employed to illustrate the load flow procedures was too small to investigate the problem of unbalances in a real sense, the sample system of Fig. 3.9 is worked out for the following simultaneous unbalances in three-phase and six-phase part of the network.

i) The loads at buses (16-18) were unbalanced to 120 percent, 100 percent and 80 percent of their base case values.

ii) The wye/delta transformer T_2 connected between buses (4-6)/(13-15) was replaced by an open delta.

Table 3.14

Summary of results showing number of iteration, computational time and the effects of starting solutions on the phase coordinate load flow

S.No.	Type of study and scheme of load flow	System size (No. of buses)	Type of starting solution	No. of iterations	CPU time in sec.
1.	Normal balanced load flow				
	(i) Scheme I	39	a	118	5.49
			b	102	5.45
	(ii) Scheme III	99	a	130	40.09
			b	98	30.09
2.	Unbalanced load flow				
	(i) With unbalances only in 3-phase part of the network (Scheme I)	39	a	120	5.50
			b	103	5.46

contd ...

S.No.	Type of study and scheme of load flow	System size (No.of buses)	Type of starting solution	No. of iterations	CPU time in sec.
(ii)	With unbalances considered only in 3-phase part of the network (Scheme III)	101	a b c	130 109 49	40.90 34.48 16.02
(iii)	With unbalances considered only in 6-phase part of the network (Scheme III)	101	a b c	127 101 54	39.88 31.97 17.54
(iv)	With unbalances considered only in 6-phase part of the network those six-phase elements where unbalances do not occur are represented by their three-phase equivalent (Scheme III)	65	a b c	72 55 31	9.87 7.70 4.63
(v)	With unbalances considered in both three-phase and six-phase part of the network (Scheme III)	103	a b c	128 111 60	40.98 35.73 19.74

iii) The off nominal tap ratio of the above transformer (T_2) was set at -0.025 on secondary side.

iv) The line conductor between buses 19 and 22 was opened at both ends. Two additional nodes 100 and 101 were introduced on which the opened conductor was assumed to be terminated.

v) The line conductor between buses 64 and 70 was opened at both ends. Two additional nodes 102 and 103 were introduced on which the opened conductor was assumed to be terminated.

vi) The three-phase/six-phase, wye/star transformer T_6 connected between buses (10-12)/[52-57] was set at -0.025 off nominal turns ratio on secondary side.

It may be noted that unbalances (i) - (iv) occur in the three-phase part and (v) and (vi) occur in six-phase part of the network. The results of load flow study are recorded in Tables 3.15 and 3.16 along with the base case solution for a comparative examination.

Discussion of Results

From a careful examination of the results in Tables 3.15 and 3.16, the following observations are made :

i) When every element of the system is balanced and the transmission lines are fully transposed, the system voltages and power flows are balanced.

ii) Under the unbalances (six in number and applied simultaneously) the maximum unbalancing in system voltages is below 4 percent whereas the considerable unbalancing exists in power flows especially in machines where it is as high as 20 percent. Although, the system voltages are under permissible limits for safe operation of the system, the negative and zero sequence quantities may pose serious problem of overheating and relay maloperation.

iii) The voltages and power flows for six-phase transmission lines follow a definite pattern where the corresponding values are identical even in unbalanced conditions for phase pairs (1,4), (3,6) and (5,2) having the similar phases of two mutually coupled three phase systems with 180° phase shift. This is consistent with the way a six-phase system has been conceptualized in this investigation.

iv) A large departure of phase angles for bus groups (4-6) and (28-30) in unbalanced conditions from the base case values is due to the change of connection of wye/delta transformer T_2 to an open delta.

v) As the system has been considered to be solidly earthed, the voltages of neutral bus bars 31,32 and 39 are zero both in magnitude and phase angle.

vi) The opened conductor terminal buses (100-103) show small voltages and power flows (only between buses 100-101) and this is because of the presence of capacitance to earth and mutual coupling of opened conductors with the other phases.

vi) The most effective starting solution for unbalanced phase coordinate load flow analysis is the base case solution and three-phase/single phase solution are the most effective for base case phase coordinate load flow as quantified in Table 3.15.

The phase coordinate load flow method provides the unbalanced voltages, line flows and MVA loadings of generators which are not revealed by normal single phase load flows. The informations obtained from the study may be utilized to find out the effects of unbalanced system operation on machines, transmission lines and other equipments. In fact, in several cases, the unbalance effect may not be especially significant as far as the system network is concerned but in terms of the individual components, it may be extremely important. In its simplest form, the method need not have the usual automatic adjustment features like balanced load flows since the majority of unbalances resulting from network impedances and loads are unaffected by these controls. Moreover, as these studies are meant only to investigate few important and complex situations and may invariably preceed several runs of

balanced load flows, a starting solution close to the required unbalanced solution may always be made available.

Table 3.15

Voltage solutions for phase coordinate load flow study (Scheme III)

Bus No.	Base case		Unbalanced case	
	$\|V\|$ (p.u.)	θ (deg.)	$\|V\|$ (p.u.)	θ (deg.)
1	1.049	-2.566	1.042	-2.993
2	1.050	-122.699	1.044	-122.613
3	1.049	117.560	1.051	117.319
4	1.046	-2.985	1.052	87.535
5	1.046	-123.052	1.047	-31.659
6	1.046	117.046	1.061	-151.805
7	1.048	-4.167	1.032	-4.781
8	1.048	-124.236	1.033	-124.432
9	1.048	115.866	1.039	115.399
10	1.039	85.929	1.037	86.005
11	1.039	-33.975	1.038	-34.781
12	1.040	-154.067	1.027	-154.259
13	1.030	85.067	1.011	85.368
14	1.031	-34.847	1.017	-36.195
15	1.031	-154.910	0.996	-155.259
16	1.034	85.046	1.010	84.950
17	1.034	-34.864	1.015	-35.840
18	1.035	-154.937	1.004	-155.172
19	1.028	84.743	1.006	84.723
20	1.029	-35.167	1.015	-36.372
21	1.029	-155.238	0.998	-155.563

contd...

Bus No.	Base case		Unbalanced case	
	|V| (p.u.)	θ (deg.)	|V| (p.u.)	θ (deg.)
22	1.029	84.794	1.018	85.094
23	1.029	-35.116	1.018	-36.332
24	1.030	-155.190	1.000	-155.477
25	1.070	0.000	1.070	0.000
26	1.070	-120.144	1.070	-120.144
27	1.070	120.144	1.070	120.144
28	1.070	-0.405	1.070	90.646
29	1.071	-120.462	1.071	-29.411
30	1.070	119.603	1.070	-149.419
31	0.000	0.000	0.000	0.000
32	0.000	0.000	0.000	0.000
33	1.051	-4.053	1.038	-4.665
34	1.052	-124.121	1.039	-124.384
35	1.051	115.975	1.044	115.501
36	1.070	-2.786	1.070	-3.221
37	1.071	-122.843	1.071	-123.278
38	1.070	117.222	1.071	116.787
39	0.000	0.000	0.000	0.000
40	1.036	84.484	1.030	85.640
41	1.037	25.492	1.018	25.246
42	1.037	-34.423	1.031	-35.404
43	1.036	-94.516	1.030	-94.359
44	1.037	-154.508	1.018	-154.755
45	1.037	145.477	1.031	144.599
46	1.033	85.244	1.026	85.470
47	1.034	25.256	1.011	25.023
48	1.034	-34.663	1.026	-35.714
49	1.033	-94.756	1.026	-94.530
50	1.034	-154.744	1.011	-154.978
51	1.034	145.337	1.026	144.289

contd...

Bus No.	Base case		Unbalanced case	
	|V| (p.u.)	θ (deg.)	|V| (p.u.)	θ (deg.)
52	1.038	85.564	1.012	85.587
53	1.039	25.573	1.003	25.374
54	1.039	-34.343	1.014	-35.212
55	1.038	-94.436	1.012	-94.412
56	1.039	-154.427	1.003	-154.627
57	1.039	145.657	1.014	144.791
58	1.036	85.413	1.011	85.392
59	1.037	25.425	1.003	25.210
60	1.037	-34.495	1.014	-35.409
61	1.036	-94.587	1.011	-94.607
62	1.037	-154.575	1.003	-154.799
63	1.037	145.505	1.014	144.593
64	1.032	84.915	1.010	84.906
65	1.033	24.932	1.002	24.655
66	1.033	-34.996	1.016	-36.084
67	1.032	-95.085	1.010	-95.137
68	1.033	-155.068	1.002	-155.337
69	1.033	145.004	1.016	143.923
70	1.031	84.875	1.007	84.770
71	1.032	24.893	1.001	24.612
72	1.032	-35.035	1.015	-36.157
73	1.031	-95.125	1.008	-95.184
74	1.032	-155.107	1.001	-155.400
75	1.032	144.965	1.015	143.841
76	1.030	84.916	1.009	85.073
77	1.030	24.936	0.997	24.601
78	1.030	-34.997	1.016	-36.314
79	1.030	-95.084	1.009	-94.923
80	1.030	-155.064	0.997	-155.401
81	1.030	145.003	1.016	143.685
82	1.030	85.985	1.009	85.033
83	1.030	24.915	0.997	24.578
84	1.030	-35.017	1.016	-36.318
85	1.030	-95.105	1.008	-94.968
86	1.030	-155.085	0.997	-155.424
87	1.030	144.983	1.016	143.682

contd ...

Bus No.	Base case		Unbalanced case	
	|V| (p.u.)	θ (deg.)	|V| (p.u.)	θ (deg.)
88	1.030	84.914	1.015	85.212
89	1.030	24.933	0.998	24.615
90	1.030	-34.998	1.018	-36.294
91	1.030	-95.068	1.015	-94.786
92	1.030	-155.067	0.998	-155.386
93	1.030	145.002	1.018	143.708
94	1.030	84.947	1.015	85.255
95	1.031	24.967	0.998	24.649
96	1.030	-34.965	1.018	-36.270
97	1.030	-95.053	1.015	-94.743
98	1.031	-155.033	0.998	-155.353
99	1.030	145.035	1.018	143.732
100	-	-	0.206	1.039
101	-	-	0.206	0.775
102	-	-	0.223	-0.348
103	-	-	0.223	-0.284

Table 3.16

Power Flows

From Bus	To Bus	Base case		Unbalanced case	
		Real power (MW)	Reactive power (MVAR)	Real power (MW)	Reactive power (MVAR)
(a) Three phase machine (generators)					
25	1	52.015	23.865	60.244	32.510
26	2	51.823	23.087	49.797	29.646
27	3	52.426	23.432	57.381	22.148
28	4	29.662	15.552	35.109	12.486
29	5	29.824	15.936	25.907	15.527
30	6	29.409	15.850	27.867	5.984
36	33	14.648	11.435	16.435	20.215
37	34	14.797	11.965	12.644	20.195
38	35	14.428	11.898	14.800	16.754

contd...

From Bus	To Bus	Base case Real power (MW)	Base case Reactive power (MVAR)	Unbalanced case Real power (MW)	Unbalanced case Reactive power (MVAR)
(b) Three phase lines					
7	33	-8.639	-9.310	-10.467	-17.379
8	34	-8.786	-9.642	-6.595	-17.456
9	35	-8.786	-9.595	-8.791	-14.168
19/100	22/101	-3.896	-0.868	-0.551	0.352
20	23	-3.961	-0.995	-6.635	-6.854
21	24	-3.824	-0.985	-3.555	-7.961
(c) Six phase lines					
40	46	9.358	2.898	7.658	6.843
41	47	9.275	3.016	9.900	9.775
42	48	9.423	3.038	12.590	7.015
43	49	9.357	2.898	7.658	6.843
44	50	9.275	3.016	9.902	9.775
45	51	9.423	3.036	12.587	7.014
52	58	7.671	1.372	8.787	-1.160
53	59	7.577	1.501	7.333	-2.525
54	60	7.740	1.527	8.596	-2.744
55	61	7.671	1.373	8.785	-1.159
56	62	7.577	1.501	7.334	-2.523
57	63	7.740	1.527	8.597	-2.744
64/102	70/103	2.731	1.648	0.000	0.000
65	71	2.723	1.655	2.796	1.730
66	72	2.734	1.657	4.052	-0.445
67	73	2.731	1.648	3.245	2.921
68	74	2.723	1.655	3.811	-1.475
69	75	2.734	1.657	4.327	-1.469
76	82	3.131	0.763	6.088	2.606
77	83	3.173	0.699	2.754	-1.885
78	84	3.098	0.694	0.678	0.868
79	85	3.131	0.763	6.091	2.602
80	86	3.172	0.700	2.750	-1.891
81	87	3.098	0.694	0.678	0.867

contd ...

From Bus	To Bus	Base case		Unbalanced case	
		Real power (MW)	Reactive power (MVAR)	Real power (MW)	Reactive power (MVAR)
88	94	-2.491	-1.394	-2.281	2.599
89	95	-2.546	-1.322	-1.776	1.494
90	96	-2.456	-1.309	-1.648	-0.413
91	97	-2.491	-1.394	-2.280	2.598
92	98	-2.546	-1.322	-1.777	1.495
93	99	-2.456	-1.309	-1.647	-0.414

3.5 CONCLUSIONS

In this chapter, the various schemes of modelling a six-phase (multi-phase) systems in balanced as well as in unbalanced conditions have been presented to carry out the load flow studies. The procedure of phase coordinate method has been particularly emphasised. Three alternative schemes have been suggested to effectively deal with the problem of unbalances either occurring in three-phase or six-phase or both three-phase and six-phase parts of the network. The procedure of modelling, and load flow analysis are demonstrated with the help of sample network calculations. Employing these techniques, three case studies are presented to bring out the relative performance of six-phase (multi-phase) systems over three-phase systems. Some of the important conclusions drawn from the study in this chapter may be summarised as;

i) The high-phase order systems indicate their potentiability to maintain better voltage magnitudes and phase angles even at higher phase loadings than that of conventional three-phase system.

ii) The impact of converting an existing double circuit three-phase line to six-phase line is to improve the voltage regulation and line efficiency, and increased transmission capability (73.2 percent). These benefits increase as more and more double circuit three-phase lines are uprated in this manner.

iii) The phase coordinate load flow Scheme III is suitable for simulating large systems as discussed in Section 3.4.3. The method of obtaining an effective starting solution has been discussed and the economics of computer time for other practical schemes are also discussed.

iv) In addition, the complete solution of a 99 bus system is presented and discussed to focus certain features of six-phase (multi-phase) system performance characteristics.

Although, the test systems considered in the present investigation are solidly earthed and all transmission networks are assumed to be completely transposed, the program developed for the purpose is not constrained by these features. Since the attempts have been mainly to develop various load flow schemes so that a six-phase (multi-phase) system and its

CHAPTER IV

FAULT ANALYSIS

4.1 INTRODUCTION

Fault analysis is one of the important studies required for the design of adequate protective schemes. Fault analysis of six-phase (multi-phase) systems has been earlier attempted by several authors [13,14,16-20] employing the transformation of actual quantities to either symmetrical or Clarke's components. In most of these studies, the main objective has been towards the derivation and basic understanding of the transformations for six-phase and also higher phase order systems. The sequence networks and their interconnections for different types of faults have been drawn for conceptual clarity. The complexity and types of fault in six-phase systems are greater than in three-phase systems. Earlier, all eleven class of faults likely to occur in six-phase transmission systems [18] were discussed and the results were summarised in tabular form. The fact that there are six-phases and each is subjected to different voltages, and a neutral connection in a six-phase system, the total number of fault combinations become as high as 120 as compared to only 11 in three-phase systems. However, there are only 23 combinations with distinct fault levels and phase interconnections and are summarised by Gauker et al [6]. In

addition, the fault analysis of a proposed six-phase line has been carried out by simplifying the rest of the three-phase network by the equivalent six-phase Thevenin's equivalents at both ends of the line by Venkata et al [20].

From practical view points, it would be desirable to incorporate proper fault impedances/admittances, interfacing transformers for six-phase conversion, and adequate representation of the rest of the three-phase network so that the performance of the overall system (six-phase and also composite three-phase and six-phase) during fault may be investigated in greater details. Suitable techniques for handling more complex faults and their combinations need to be developed since the transformation methods become unwieldy for such cases involving unbalanced network.

In this chapter, generalised procedures for a six-phase and also a composite three-phase and six-phase system are developed employing transformation as well as phase coordinate methods. Firstly, in Section 4.2.1, the three-phase symmetrical component technique employing bus impedance description of the network [37] is generalised to six-phase systems. Next in Section 4.2.2, the method of phase coordinates is presented to investigate the problem of unbalances and simultaneous faults. The numerical examples and case studies results are presented in Section 4.3. The chapter concludes with the discussion of salient features of the study in Section 4.4.

4.2 SYMMETRICAL AND CLARKE'S COMPONENTS TRANSFORMATIONS FOR MULTI-PHASE POWER NETWORKS

Multi-phase power system networks which are inherently symmetric, can be broadly classified into two categories : (1) rotating elements, and (2) stationary elements.

4.2.1 Rotating Elements

Multi-phase power system networks which possess symmetry operations involving proper rotations are known as rotating elements. By exploiting these symmetries and using representation theory of finite groups [20], block diagonalising transformation matrices can be constructed. Since symmetry elements of such network constitute a regular Abelian (i.e. commutative) cyclic group, block diagonalising transformations can also be developed using eigenvalue approach. The network equation for a six-phase element p-q in this case will be :

$$V^6_{pq} = Z^6_{pq}\ i^6_{pq} \tag{4.1}$$

where V^6_{pq}, Z^6_{pq} and i^6_{pq} are in phase variable form and the phase sequence being a,b,c,d, e and f respectively. The coefficient matrix Z^6_{pq} will be cyclic. The transformation matrix T^6_S to relate the phase quantities to the component quantities [21] is of the formgiven by eqn. (2.3). Since $[T^{6*}_S]^T\ [T^6_S] = U^6$, the transformation, which is linear, complex and power invariant, is the familiar symmetrical component transformation for six-phase systems.

4.2.2 Stationary Element

Multi-phase system networks which possess, in addition to the symmetry operations of proper rotations, reflection symmetries also, are known as stationary elements; as typical example is that of transposed transmission lines. These symmetry operations constitute a group, and thus by using representation theory of finite groups, block diagonalising transformation matrices can be constructed. If a complex basis is chosen for the representation, the complex transformation T_S^6 is obtained, but if a real basis is chosen, the transformation matrix T_C^6 with real elements is obtained [21] as shown,

$$T_C^6 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & \sqrt{3}/2 & 1/\sqrt{2} & 1 & -\sqrt{3}/2 & -1/\sqrt{2} \\ 1 & \sqrt{3}/2 & 1/\sqrt{2} & -1 & \sqrt{3}/2 & 1/\sqrt{2} \\ 1 & 0 & -\sqrt{2} & 1 & 0 & \sqrt{2} \\ 1 & 0 & -\sqrt{2} & -1 & 0 & -\sqrt{2} \\ 1 & -\sqrt{3}/2 & 1/\sqrt{2} & 1 & \sqrt{3}/2 & -1/\sqrt{2} \\ 1 & -\sqrt{3}/2 & 1/\sqrt{2} & -1 & -\sqrt{3}/2 & 1/\sqrt{2} \end{bmatrix} \qquad (4.2)$$

The matrix T_C^6 is orthogonal $[T_C^6]^T[T_C^6] = U^6$, and is linear, power invariant real transformation known as Clarke's component transformation for six-phase system.

Various component transformations for diagonalising the phase impedance/admittance matrix of multi-phase systems, particularly the six-phase systems, are available in the

literature [12-17, 19, 21,22] out of which those proposed by Willems [16] and Singh et al [21,22] need special mention here. The symmetrical and Clarke's component transformations derived by Willems [16] conceptualises a six-phase system as two coupled three-phase systems. It is pointed out [16] that similar transformations may be obtained for higher order systems where the phase order is a multiple of three. Although the Clarke's component transformation [16] is power variant, it is applicable to systems with cyclic but non-complete transposition of line as well. If a complete transposition of conductors (though physically too difficult to realise but a reasonable assumption for transmission network) is made, then Clarke's component transformations similar to (4.2) can be developed for higher order multi-phase system including those with phase order not necessarily a multiple of three, following the firm mathematical group theoretic considerations by Singh et al. [22].

4.3 FAULT ANALYSIS

The analysis of faults in six-phase system is much more complicated than in three phase system. Because of the six-phases subjected to different voltages and a neutral connection, the total number of fault combinations are much more than the three phase case. To get an insight into the problem, a table showing the total number of faults and their significant combinations is reproduced from [6,20] for ready reference in Table 4.1.

Table 4.1

Types of fault and the number of phase and/or neutral combinations

Fault type	Six-phase system			Three-phase system		
	Total No.of combi-nations	Signi-ficant No.of combi-nations	Faulted phases for significant com-binations	Total No.of combi-nations	Signi-ficant No.of combi-nations	Faulted phases significant com-binations
Phase to neutral	1	1	a-n	1	1	a-n
Two phase to neutral	15	3	a-d-n, b-f-n, b-c-n	3	1	b-c-n
Two phase	15	3	a-d, b-f, b-c	3	1	b-c
Three phase to neutral	20	3	b-d-f-n, a-b-d-n, a-b-f-n	1	1	a-b-c-n
Three phase	20	3	b-d-f, a-b-d, a-b-f	1	1	a-b-c
Four phase to neutral	15	3	b-c-e-f-n, a-b-c-d-n, a-b-d-f-n			
Four phase	15	3	b-c-e-f, a-b-c-d, a-b-d-f			
Five phase to neutral	6	1	b-c-d-e-f-n			
Five phase	6	1	b-c-d-e-f			
Six-phase to neutral	1	1	a-b-c-d-e-f-n			
Six phase	1	1	a-b-c-d-e-f			
Total	120	23		11	5	

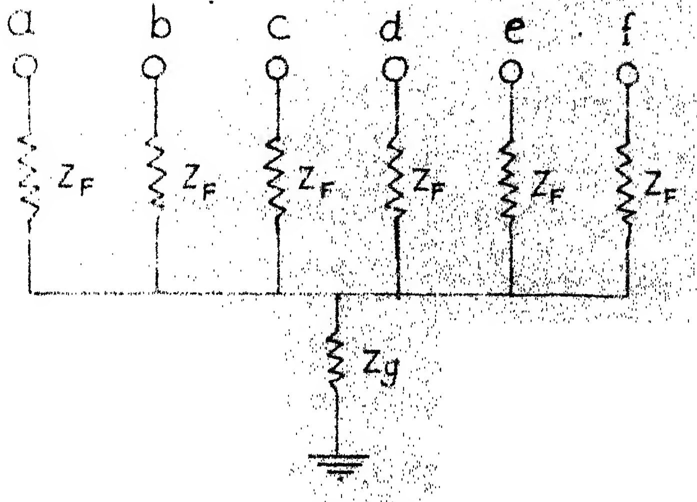


FIG. 4.1 SCHEMATIC REPRESENTATION OF SIX-PHASE TO GROUND (LLLLLL-G) FAULT IN 9 SIX-PHASE SYSTEM

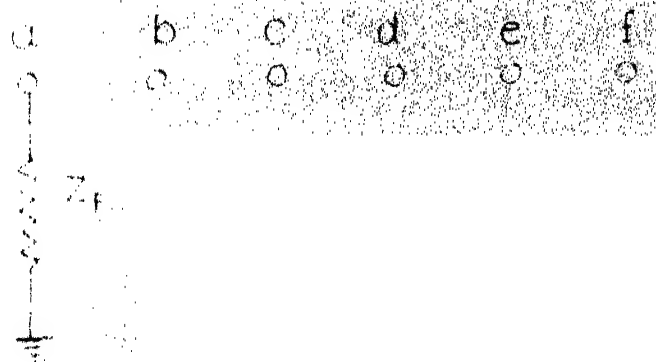


FIG. 4.2 SCHEMATIC REPRESENTATION OF ONE-PHASE FAULT IN A SIX-PHASE SYSTEM

Any comprehensive fault analysis must take into account all the significant combinations of faults which will have important bearing on the design of adequate protective schemes for six-phase system.

As pointed out earlier, the existing techniques of fault analysis for six-phase system [13-20] employing one or the other versions of symmetrical and Clarke's component transformations have been applied to a six-phase line considered in isolation (conceptually) or by focussing attention on a specific line in the entire network with the rest of the network represented by Thevenin's equivalents on either ends of the line. In doing so, the information regarding the other points of the system during fault is suppressed. Such a drastic simplification does not seem to be necessary. In this section, the generalised procedure for fault analysis without requiring such simplification, is developed employing transformation and phase coordinate representation. With the developments to follow, a six-phase and also a composite three-phase and six-phase system can be analysed for faults in the same manner as the conventional three-phase system.

4.3.1 Transformation Method

The bus impedance formulation of fault analysis problem [37] provides an efficient means of determining short circuit currents and voltages with few arithmetic operations involving

only the related portion of the bus impedance matrix. Both methods employing actual phase quantities and transformed unbalanced phase quantities may be applied to obtain the solution. It is to be noted that the network is considered to be balanced with all other usual assumptions and use of Thevenin's theorem is made for short circuit calculations. Assuming that the system to be treated here is either a completely six-phase system or is an equivalent six-phase system of an integrated three-phase and six-phase system, a straight forward extension of the technique [37] yields all the required informations.

The equations for fault currents and voltages with fault at a six-phase bus k involving fault impedance/admittance matrix Z_F^6/Y_F^6 are given by

$$I^6_{k(F)} = (Z_F^6+Z_{kk}^6)^{-1}\ E^6_{k(o)};\quad I^6_{k(F)} = Y_F^6(U^6+Z_{kk}^6)^{-1}\ E^6_{k(o)} \quad (4.3)$$

$$E^6_{k(F)} = Z_F^6(Z_F^6+Z_{kk}^6)^{-1}\ E^6_{k(o)};\ E^6_{k(F)}= (U^6+Z_{kk}^6Y_F^6)^{-1}\ E^6_{k(o)} \quad (4.4)$$

The post fault voltages at buses other than k are given by

$$E^6_{i(F)} = E^6_{i(o)}-Z^6_{ik}(Z_F^6+Z_{kk}^6)^{-1}\ E^6_{k(o)};$$

$$E^6_{i(F)} = E^6_{i(o)}-Z^6_{ik}\ Y_F^6(U^6+Z_{kk}^6Y_F^6)^{-1}E^6_{k(o)}\ ;\ i \neq k,\ i = 1,2,\ \ldots N \quad (4.5)$$

Eqns. (4.3 - 4.5) are applicable in general to both methods using phase and transformed variables, but the simplifications introduced by transformations make the latter more attractive and well accepted method of analysis. As an illustration, the cases of single phase to ground (L-G) and six-phase to ground (LLLLLL-G) faults are discussed employing symmetrical component transformations.

Single phase-to ground (L-G) fault

The required fault admittance matrix $Y_F^{S,6}$ is given by eqn. (4.7) in Table 4.2. Making use of (4.3), the fault current in terms of sequence components is obtained as,

$$I_{k(F)}^{(1)} = \frac{\sqrt{6}\, E_{K(0)}}{Z_{kk}^{(o)}+6Z_F+Z_{kk}^{(1)}+Z_{kk}^{(2)}+Z_{kk}^{(3)}+Z_{kk}^{(4)}+Z_{kk}^{(5)}} = I_{k(F)}^{(o)} = I_{k(F)}^{(2)}$$

$$= I_{k(F)}^{(3)} = I_{k(F)}^{(4)} = I_{k(F)}^{(5)} \qquad (4.8)$$

The fault current expressed in phase variables is given by

$$I_{k(F)}^{a} = \sqrt{6}\, I_{k(F)}^{(1)} \text{ , and}$$

$$I_{k(F)}^{b} = I_{k(F)}^{c} = I_{k(F)}^{d} = I_{k(F)}^{e} = I_{k(F)}^{f} = 0 \qquad (4.9)$$

Six-phase to ground (LLLLLL-G) fault

Making use of the required fault impedance matrix given by (4.6) in Table 4.2 the fault current obtained vide eqn. (4.3) in sequence components is given by

Table 4.2

Fault impedance and admittance matrices for six-phase systems

S.No.	Type of fault	Phase components		Symmetrical components	
		Impedance Matrix	Admittance Matrix	Impedance Matrix	Admittance Matrix
1.	Six-phase to ground (LLLLLL-G) fault Fig. 4.1	$Z_F^{p,6}=A$ where $A \in a_{ij}=z_F+z_g \;\forall\, i=j$; $=z_g \;\forall\, i\neq j$; $i=1,2,..,6$; $j=1,2,..,6$	$Y_F^{p,6}=B$ where $B \in b_{ij}=(y_o+5y_F)/6 \;\forall\, i=j$; $=(y_o-y_F)/6 \;\forall\, i\neq j$; $y_o=1/(Z_F+6Z_g)$	$Z_F^{s,6}=\mathrm{Diag}[z_F+6Z_g, z_F, z_F, z_F, z_F, z_F]$	$Y_F^{s,6}=\mathrm{Diag}[y_o, y_F, y_F, y_F, y_F, y$ (4.6)
2.	Single-phase to ground (L-G) fault Fig.4.2	$Z_F^{p,6}=\mathrm{Diag}[z_F,\infty,\infty,\infty,\infty,\infty]$	$Y_F^{p,6}=\mathrm{Diag}[y_F,o,o,o,o,o]$	Not defined	$Y_F^{s,6}=\frac{y_F}{6}[C]$ where $C \in c_{ij}=1$; $i=1,2,...,6$; $j=1,2,...,6$ (4.7)

where superscripts p,6 denote phase frame of reference and number of phases, and s denotes the symmetrical component respectively.

$$I^{(1)}_{k(F)} = \frac{\sqrt{6}E_{K(o)}}{Z_F + Z^{(1)}_{kk}} \quad ; \quad I^{(o)}_{k(F)} = I^{(2)}_{k(F)} = I^{(3)}_{k(F)} = I^{(4)}_{k(F)} = I^{(5)}_{k(F)} = 0 \tag{4.10}$$

The fault current expressed in phase variables is given by

$$I^{a}_{k(F)} = \frac{1}{\sqrt{6}} I^{(1)}_{K(F)}; \; I^{b}_{k(F)} = \alpha^* I^{a}_{k(F)} \; ; \; I^{c}_{k(F)} = -\alpha I^{a}_{k(F)}; \; I^{d}_{k(F)} = -I^{a}_{k(F)}$$

$$I^{e}_{k(F)} = -\alpha^* \; I^{a}_{k(F)}; \; I^{f}_{k(F)} = \alpha \; I^{a}_{k(F)} \tag{4.11}$$

It has been verified that the value of current as given by (4.9) and (4.11)agree with those obtained employing different transformations proposed in the literature [14,16,17,18] vide the basic equation (4.3).

Algorithm

The following steps are required to carry out the fault analysis :

i) Perform the equivalent single phase load flow to determine the prefault voltages $E_{k(o)}$.

ii) Form the sequence component bus impedance matrices. For a completely six-phase systems, only the formation of zero and positive sequence bus impedance matrices are required because of the assumption of fully transposed transmission networks.

iii) Calculate and store the sequence component fault impedance/admittance matrices. It may be noted that to carry out the fault analysis for all significant combinations twenty three such matrices will be needed.

iv) Determine the fault current using (4.8) - (4.11)as appropriate.

v) Calculate the post-fault voltages, currents and power flows in the network as usual.

4.3.2 Phase Coordinate Method

The method of phase coordinates is more flexible and powerful technique for fault analysis especially where the transformation techniques become unwieldy or too cumbersome to apply. Moreover, all types of series, shunt and simultaneous faults can be analysed with the same ease as the balanced faults. The theory and application of phase coordinate methods are fairly well developed for fault analysis of three-phase systems and several significant papers have been reported in the literature [23,25-28,30,37]. However, the phase coordinate techniques have not been exploited to deal with six-phase (multi-phase) system although its potentiality to tackle several uncommon faults have been recognised [20]. An extension of the phase coordinate techniques to six-phase systems to deal with complex faults and unbalances in the network is carried out as follows :

The procedure is much similar to load flow except the following changes.

i) The system nodal admittance matrix Y is modified to Y' to incorporate the changes in machine model (Chapter 2, eqn. (2.1)) and load representations. The machine representation (2.1), is modified by deleting the equation relating the total complex power S and phase-a induced emf (E_a) since S becomes an unknown variable in this case. All static loads are converted to equivalent admittances and are added to the respective diagonal terms of Y. With these changes, the performance of the system is described by,

$$[Y']V = I' \tag{4.12}$$

ii) The solution of (4.12) for various faults proceeds as follows [25]

a) Ground faults (bolted faults)

If a single phase to ground fault occurs at bus k then

$$V_k = 0 \quad \text{and} \quad I_k = \text{unknown} \tag{4.13}$$

Equation (4.12) is solved under the constraint (4.13) for (N-1) voltages V_i, $i = 1, \ldots, N$, $i \neq k$.

The value of current at bus k is then found as,

$$I_k'' = \sum Y_{km}' V_m$$

$$= I_k' + I_{SC} \qquad (4.14)$$

where I_K' is the prefault value in (4.12) and I_{SC} is the current in the short circuit connection to earth. For faults involving more phases than one, the procedure is similar except the constraints in (4.13) equal the number of phases being faulty.

(b) Ground fault via impedance

If the fault at bus k involves an impedance $z_{k(F)}$, then the corresponding admittance $y_{k(F)}$ $(= Z_{k(F)}^{-1})$ is added to the principal diagonal element Y_{kk}' and with this change, eqn. (4.12) is solved for all voltages including the kth bus. The fault current is obtained from

$$I_{SC} = y_{k(F)} V_k \qquad (4.15)$$

The case of more phases than one faulted together is taken care in a similar manner.

(c) Multi-phase faults

For a phase to phase fault involving buses i and j, the constraints are

$$V_j = V_k \;; \quad I_j, I_k = \text{unknown} \qquad (4.16)$$

Eqn. (4.12) is solved iteratively with the constraints (4.16), the current at bus k is obtained as in (4.14) and is related to the prefault current I_k' and total short circuit currents as :

$$I_k'' = I_k' + \sum I_{SC} \qquad (4.17)$$

where $\sum I_{SC}$ is the total short circuit current injected into the bus bar k from all other buses short circuited to bus k.

The procedure is extended to any number of phases and the case of short circuit involving impedances.

(d) Conductor and Phase Openings

The conductor and phase openings on six-phase transmission networks for fault analysis can be simulated in the same manner as discussed for load flow studies (Chapter III). The required modification of the base case matrix for a conductor opening given by eqns. (3.22) - (3.25) may be extended to the cases involving several conductor and phase openings without any restriction on their number and geographical locations by a mere repetition of the algorithm.

(e) Simultaneous Faults

When number of faults (viz. series, shunt and their combinations) occur simultaneously in three phase, or six-phase or both three-phase and six-phase part of the network, the situation can be handled conveniently by phase coordinate

method without calling for extra care. The system performance eqn. (4.12) with appropriate constraints corresponding to the various faults (considered simultaneously) and suitable modifications to the original network matrix (if needed) are solved. The short circuit currents are obtained using equations similar to (4.14) - (4.15) as appropriate. The procedure is applicable without any restrictions on the number, type and geographical locations of the faults.

Further, the various Schemes (I,II and III) and their economics pertaining to computational time and storage requirements for load flow studies (Chapter III, Section 3.3.2) are applicable to fault analysis as well.

4.4 NUMERICAL EXAMPLES

The sample system of Fig. 3.6 is employed to check the validity of the program and to illustrate the procedures.

4.4.1 Comparison of Transformation and Phase Coordinate Methods

A single phase to ground (L-G)/six-phase to ground (LLLLLL-G) fault is applied at bus 6 and the results for system configurations A,B and C employing symmetrical component transformation and phase coordinate method are presented in Table 4.3.

Table 4.3

Results of certain ground faults on sample network (Fig. 3.6) at bus 6

(All figures in p.u.)

Type of fault	Method of analysis	System configuration					
		A		B		C	
		current		current		current	
		Mag.	Ang.	Mag.	Ang.	Mag.	Ang.
L-G	SCM	2.5	88.1	5.1	86.5	2.5	88.1
	PCM	2.7	99.9	5.5	91.5	3.0	99.9
LLLLLL-G	SCM	3.7	87.2	2.8	86.4	3.5	81.0
	PCM	3.8	93.4	2.8	92.4	3.8	94.7

where SCM - symmetrical component method

PCM - phase coordinate method

4.4.2 Faults via Impedances

Three types of faults viz. single phase to ground (L-G), three phase to ground (LLL-G) and phase to phase short circuit all through an impedance ($Z_F = 0.0 + j0.1$) are applied separately on the sample system(Fig.3.7). The method of phase coordinate is used to obtain the solution in each case. The results including the buses involved in fault are presented in Table 4.4.

Table 4.4

Results of certain faults via impedance (Z_F = 0.0 + j0.1) on sample network (Fig.3.7)

S.No.	Type of fault	Faults phases	Fault current	
			Mag (p.u.)	Angle(deg.
1	L-G	16	3.38	-93.428
2	LLL-G	16	3.28	-92.930
		17	3.28	147.080
		18	3.28	27.120
3	L-L	10-11	2.29	-31.870
		10-13	2.59	-86.500

4.4.3 Ground Short Circuits of a Six-phase Transmission Line

A detailed investigation of ground short circuits of a six-phase transmission line connected to three-phase network via wye/star transformer is carried out for its various significant combinations. The sample system chosen for the study is that of Fig. 3.7. The details of the results are presented in Table 4.5.

4.4.4 Simultaneous Faults

In order to demonstrate the capability of phase-coordinate method to simulate simultaneous faults, the network of Fig.3.9

Table 4.5

Summary of results for ground faults at buses(10-15) of network in Fig. 3.7

S.No.	Type of fault	Buses involved	Fault current	
			Mag. (p.u.)	Angle (deg)
1	L-G	10	5.5	91.2
2	LL-G			
(a)		10 11	5.4 5.3	94.3 28.3
(b)		10 12	5.4 5.4	88.5 -25.8
(c)		10 13	2.9 2.9	92.0 -87.9
3	LLL-G			
(a)		10 11 12	5.2 5.2 5.2	91.8 31.5 -28.6
(b)		10 12 14	5.2 5.2 5.2	91.6 -28.3 -148.3
(c)		10 11 14	5.4 2.8 2.9	94.4 27.3 -148.8
4	LLLL-G			
(a)		10 11 13 14	2.9 2.8 2.9 2.8	95.2 29.0 -84.7 -150.9

contd ...

S.No.	Type of fault	Buses involved	Fault current Mag. (p.u.)	Fault current Angle (deg)
(b)		10 11 12 13	2.9 2.8 2.9 2.8	95.2 29.0 -84.7 -150.9
(c)		10 11 13 15	2.7 5.2 2.9 5.2	92.2 31.3 -87.3 151.9
5	LLLLL-G	10 11 12 13 14	2.8 2.7 5.2 2.7 2.8	94.3 33.7 -28.4 -89.1 -149.2
6	LLLLLL-G	10 11 12 13 14 15	2.8 2.8 2.8 2.8 2.8 2.8	92.4 32.4 27.6 -87.5 -147.5 152.3

is employed. In the investigation only lines L1 and L2 are represented retaining their identities. The rest of the six phase lines are represented by their three phase equivalents resulting in total number of buses equal to 63 as against originally 99. The phase conductor between buses 13 and 22 was opened at both ends. Two simultaneous ground faults (i) single line to ground at bus 19, and (ii) three line to ground fault at buses 58,59 and 60 were applied. It may be noted that this situation might occur during a cross country faults. The fault currents, and the voltage conditions during fault are recorded in Table 4.6 and Table 4.7 respectively.

Table 4.6

Fault currents during simultaneous faults

Fault type	Bus No.	Fault current	
		Mag. (p.u.)	Angle (deg.)
Open conductors	13	-	-
	19	-	-
L-G	19	6.441	175.47
LLL-G	58	3.074	-176.71
	59	8.058	116.97
	60	8.066	56.59

Table 4.7

Voltage magnitudes and angles during simultaneous faults

Bus No No.	Voltage Mag. (p.u.)	Voltage Angle (deg)	Bus No.	Voltage Mag. (p.u.)	Voltage Angle (deg)
1	0.707	-1.55	31	0.000	0.00
2	0.668	-123.68	32	0.000	0.00
3	0.668	120.44	33	0.718	-5.99
4	0.669	-4.15	34	0.702	-127.23
5	0.634	-126.21	35	0.698	114.57
6	0.633	117.75	36	1.070	-5.24
7	0.649	-4.78	37	1.071	-125.30
8	0.630	-126.35	38	1.070	114.76
9	0.627	116.08	39	0.000	0.00
10	0.415	85.94	40	0.350	84.55
11	0.491	-30.38	41	0.421	22.82
12	0.494	-156.42	42	0.418	-31.46
13	0.281	84.91	43	0.350	-95.45
14	0.376	-31.87	44	0.421	-157.16
15	0.379	-157.49	45	0.418	148.56
16	0.257	82.90	46	0.322	84.88
17	0.310	-32.49	47	0.388	23.46
18	0.315	-157.45	48	0.385	-31.00
19	0.000	0.00	49	0.322	-95.11
20	0.252	-33.07	50	0.388	-156.53
21	0.255	-153.24	51	0.385	149.01
22	0.079	78.39	52	0.101	65.79
23	0.312	-33.58	53	0.142	16.77
24	0.315	-156.51	54	0.133	-30.76
25	1.070	0.00	55	0.111	-96.12
26	1.070	-120.14	56	0.178	-157.55
27	1.070	120.14	57	0.178	143.73
28	1.070	-3.90			
29	1.071	-123.96			
30	1.071	116.10			

contd...

Bus No.	Voltage Mag. (p.u.)	Voltage Angle (deg)	Bus No.	Voltage Mag. (p.u.)	Voltage Angle (deg)
58	0.000	0.000	62	0.178	-156.63
59	0.000	0.000	63	0.171	151.69
60	0.000	0.000	64	0.222	-2.229
61	0.068	-111.320	65	0.222	1.11

Discussion of Results

From the results of fault analysis presented in Tables 4.3 - 4.7, the following observations are made :

i) The phase coordinate and symmetrical component methods both can be applied for the analysis of faults. The phase coordinate method gives accurate results. However, the results obtained (Table 4.3) by the two methods are in good agreement. A little discrepancy in the results by transformation method is due to the usual assumptions made.

ii) The performance of a completely six-phase system during fault is similar to a three-phase system (Table 4.3). However, a composite three-phase and six-phase system shows a performance during ground short circuits which is different from other system configurations (Table 4.5). It is seen that when only one phase conductor of the six-phase line, out of two phases having 180° phase displacement is involved, the ground fault current magnitude is nearly double than that when both

phases are involved. This is observed in all significant combinations presented in Table 4.5. This situation is caused by the presence of the three-phase/six-phase transformer which has one phase winding on primary side corresponding to two-phase windings on secondary side with 180^{0} phase shift and located on the same core. The effect of the transformer on the performance of the system on three phase side (of a wye/star transformer) is to yield more current for single phase to ground fault than that for three phase fault. However, such a situation does not occur in the three-phase part of the network far away from the three-phase/six-phase transformer.

iii) The application of phase coordinate method for the analysis of simultaneous faults is demonstrated by working out the sample network of Fig. 3.9 where lines L3, L5 and L6 are represented by their three-phase equivalents thus reducing the size of the system from 99 buses to 63 buses only. It can be observed from Table 4.3 that the effect of simultaneous faults is to cause, in general, a distribution of low bus voltages. As expected, the buses adjacent to faults and corresponding to faulty phases are characterised by relatively lower voltage magnitudes.

4.5 CONCLUSIONS

In this chapter, the complexity of faults in six-phase systems and the techniques for their analysis have been discussed.

Both the methods employing symmetrical component transformation and phase coordinate representation have been developed. A straight forward extension of three-phase technique employing bus impedance description of the network to six-phase system eliminates the limitations of several existing techniques. The phase coordinate method which is most suitable for unbalanced network conditions and simultaneous faults has been employed to analyse six-phase and also a composite three-phase and six-phase systems. Numerical examples are worked out to demonstrate the procedure.

One of the important aspect of the study has been, the detailed investigation of all significant combinations of ground short circuits of a six-phase transmission line connected to the three-phase network via three-phase/six-phase, wye/star transformer. The study reveals a performance where the phase currents in six-phase to grounds faults are lesser in magnitudes than in single phase to ground fault. This type of situation which do not occur in a completely six-phase or three phase system, may have important bearing on protective hardware schemes and their ratings. The simultaneous fault investigation on a large system is demonstrated by considering an open phase conductor and cross country faults. The resulting lower bus voltage distribution as obtained, are shown in Table 4.7 for a more vivid examination.

CHAPTER V

TRANSIENT STABILITY STUDIES

5.1 INTRODUCTION

This chapter is devoted to the investigation of transient stability problem of six-phase (multi-phase) power systems. Transient stability study forms one of the most important aspects in planning and operation of power systems. In view of the fact that several utilities are examining the six-phase transmission as alternative to double circuit three-phase and/or considering a long-range change from conventional systems to multi-phase systems, the performance of multi-phase system and their addition to existing system must be investigated under all conceivable disturbances. The sudden and large disturbances arising out of fault followed by switching operations, sudden rejection of load or generation or loss of excitation etc. are most severe ones and are usually considered in transient stability studies. Determination of critical clearing time is one of the important facets of transient stability investigation, since such information enables the setting of protective relays.

Although, the literature on transient stability studies of conventional three-phase system is well documented [37-39, 42-43], the multi-phase systems have not received much

attention, possibly, due to lack of suitable mathematical models and the necessary data needed for such investigations. In the present chapter, the transient stability investigations for a six-phase and also for a composite three-phase and six-phase systems are carried out by incorporating the representations of multi-phase elements (developed in Chapter 2) by modifying/extending the existing three-phase techniques. A simple approach of alternate steady-state solution and numerical integration [37,38] is adopted for both methods using single phase equivalent as well as phase coordinate representation. The approach retains the information regarding bus voltages as function of time thus permitting a clear view of the voltage status of the system at every instant of time. A sample system is worked out to illustrate the procedure and conduct several case studies to focus the relative performance of multi-phase systems over the conventional three-phase systems.

5.2 TRANSIENT STABILITY SIMULATION PROCEDURES

There are two methods of simulation of transient stability behaviours of conventional three-phase systems currently in vogue [38]. The first method employs alternate computation cycles of the differential and network performance equations. The second methods calls for straightforward numerical integration of swing equations (employing reduced order admittance matrices of the system during

faulted and post fault stages, and general expression for electrical powers of generators). Although, the second method is quite fast as compared to the first one, the ability of the first method to provide bus voltage status of the system as a function of time, makes it, particularly useful for the present investigation. A brief discussion of the method employing equivalent single phase as well as phase coordinate description modified/extended for six-phase and also for a composite three-phase and six-phase system, follows before the presentation and discussion of results for various cases.

5.2.1 Equivalent Single Phase Procedure

In this method, the six-phase (multi-phase) elements of the network including the three-phase elements are represented by their single-phase equivalents. The stability investigations are carried out as usual by alternate computation cycles of the differential and network performance equations [38]. Some pertinent details of representation and formulation of the procedure are given as under.

i) Machine representation : A synchronous machine is represented by a voltage source, in back of transient reactance, that is constant in magnitude but changes its angular position, for the purpose of this study.

ii) Load Representation : All loads other than motors are represented by static admittances from bus to ground.

iii). Network Representation : The network performance equations employing equivalent single phase representation of various elements are described as

$$YV = I \tag{5.1}$$

The solution of equation (5.1) is obtained by Gauss-Siedel Iterative technique.

iv) Swing Equations : The swing equations with simplified machine model employed in this investigation are :

$$\begin{aligned} \frac{d\delta_i}{dt} &= \omega_i(t) - 2\pi f \\ \frac{d\omega_i}{dt} &= \frac{\pi f}{H_i} \left(P_{mi} - P_{ei(t)}\right) , \; i = 1,2, .,m \end{aligned} \tag{5.2}$$

where m is the number of machines.

In equation (5.2), the effect of damping and governor actions are neglected. The solution of (5.2) is obtained by the modified Euler's method.

5.2.2 Phase Coordinate Procedure

Although, the application of phase coordinate method may be questionable for extensive simulations of large systems on account of prohibitive memory and computational

cost; its ability to render complete network voltage status during every step of integration may be valuable in few important and complex situations. In addition, the various kinds of faults especially unbalanced ones and also disturbances can be simulated in phase coordinate method with more ease and flexibility than in other methods. Since only a preliminary examination of transient stability problems of six-phase system is intended here, the method is applied to work out certain cases of faults and their effects on the stability performance of the sample system of Fig. 5.1.

The phase coordinate method is similar to the single phase procedure except that the network performance equations are solved in phase coordinates. The swing equations of machines are written employing only the phase-a angles. This is because of the fact that the balanced design of machine is assumed implying balanced internally induced emfs. Therefore, there is no advantage in integrating all the internal phase angles of a machine since no extra information is obtained.

5.3 NUMERICAL EXAMPLES AND CASE STUDIES

The resultsof transient stability investigations and the various case studies are illustrated with the help of a sample system described in 5.3.1.

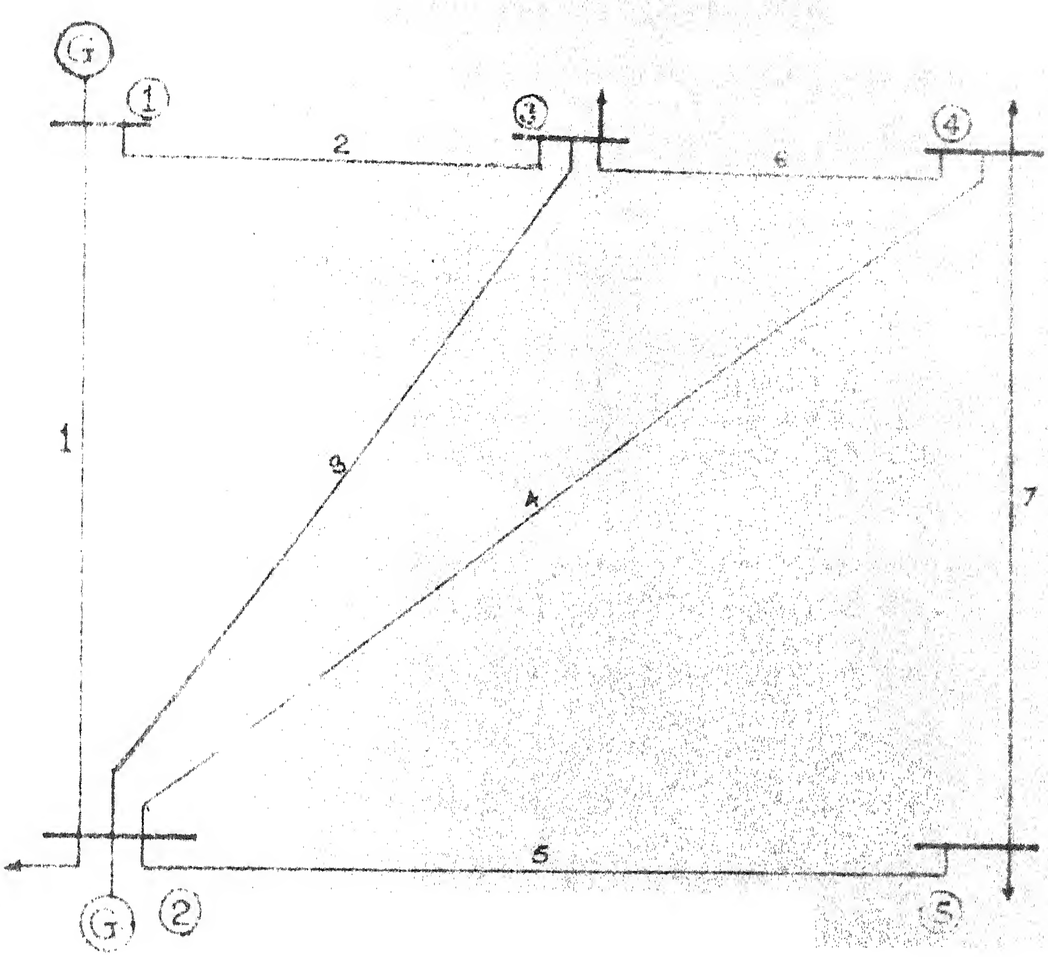


FIG. 5.1 SINGLE LINE DIAGRAM OF A SAMPLE SYSTEM FOR TRANSIENT STABILITY INVESTIGATION

5.3.1 Sample System

Fig. 5.1 shows the single line diagram of a sample system [37] chosen for illustration of transient stability investigation. The circuit characteristics, load flow and machine data are given in Tables 5.1, 5.2 and 5.3 respectively.

Table 5.1

Circuit characteristics on 100 MVA base

Line No.	Bus Code From bus p	To bus q	Impedance Z_{pq}	Line Charging $y'_{pq}/2$
1	1	2	0.02 + j0.06	0.0 + j0.030
2	1	3	0.08 + j0.24	0.0 + j0.025
3	2	3	0.06 + j0.18	0.0 + j0.020
4	2	4	0.06 + j0.18	0.0 + j0.020
5	2	5	0.04 + j0.12	0.0 + j0.015
6	3	4	0.01 + j0.03	0.0 + j0.010
7	4	5	0.08 + j0.24	0.0 + j0.025

5.3.2 Numerical Examples and Case Studies

The solution of transient stability problems of six-phase (multi-phase) systems are obtained by applying both the methods employing single phase equivalent as well as phase coordinate representation. The illustration of procedures and study of performance of multi-phase system is carried out with the help of following cases.

Table 5.2

Load flow data for sample system of Fig. 5.1

Bus Code p	Assumed bus voltage	Generation Megawatts	Generation Megavars	Load Megawatts	Load Megavars
1	1.06 + j0.0	0	0	0	0
2	1.00 + j0.0	40	30	20	10
3	1.00 + j0.0	0	0	45	15
4	1.00 + j0.0	0	0	40	5
5	1.00 + j0.0	0	0	60	10

Table 5.3

Inertia constants and direct axis transient reactances for generators of sample system of Fig. 5.1

Machine No.	Bus Code p - i	Inertia Constant H	Direct axis transient reactance x'_d
1	1 - 6	50.0	0.25
2	2 - 7	1.0	1.50

I. Performance of six-phase (multi-phase) systems

In order to obtain a relative idea of performance of high phase order systems over the conventional three-phase system, the sample system of Fig. 5.1 is analysed for the following three configurations.

i) Original three-phase system : The details are given vide Tables 5.1 - 5.3.

ii) A composite three-phase and six-phase system : Line 5 of the sample system (Fig. 5.1) is converted to a six-phase line employing two three-phase/six-phase; wye/star transformers to connect the line to three-phase buses 2 and 5. The transformer leakage impedances are assumed to be 8% based on their own ratings. In uprating the line 5 to six-phase, the line to line (adjacent) voltage is assumed to be the same in both cases.

iii) A completely six-phase system : In addition to line 5 which is operating as a six-phase line as given in (ii) all other lines are assumed to be replaced by six-phase lines (with the phase to ground voltage equal to that in three-phase case) characterised by the same per phase values of parameters. Generators 1 and 2 are also replaced by six-phase generators with the same per phase values of different parameters as that for three-phase system.

An all phases to ground fault is considered to occur at bus 2 and the fault is cleared after 0.1 sec. by the removal of the fault. The equivalent single phase procedure is employed to obtain the solution for all the three configurations. The network prefault conditions are given in Table 5.4. In order to obtain a qualitative and quantitative idea of the performance of multi-phase systems, the variation of internal machine angles δ_1, δ_2, their difference $(\delta_2-\delta_1)$, and power delivered P_{e1} and P_{e2} are plotted as functions of time in Fig. 5.2 (curves a-e).

Table 5.4

Prefault conditions of sample system (Fig. 5.1) for all three configurations with same loading conditions

Bus No.	Type	System Configurations					
		I		II		III	
		Voltage $\lvert V \rvert$ (p.u.)	Angle (deg)	Voltage $\lvert V \rvert$ (p.u.)	Angle (deg)	Voltage $\lvert V \rvert$ (p.u.)	Angle (deg)
1	Slack	1.060	0.000	1.06	0.000	1.06	0.000
2	P,Q	1.047	-2.805	1.050	-2.829	1.060	-1.481
3	P,Q	1.024	-4.995	1.029	-5.008	1.052	-2.570
4	P,Q	1.024	-5.327	1.029	-5.345	1.052	-2.738
5	P,Q	1.018	-6.149	1.034	-6.229	1.049	-3.129

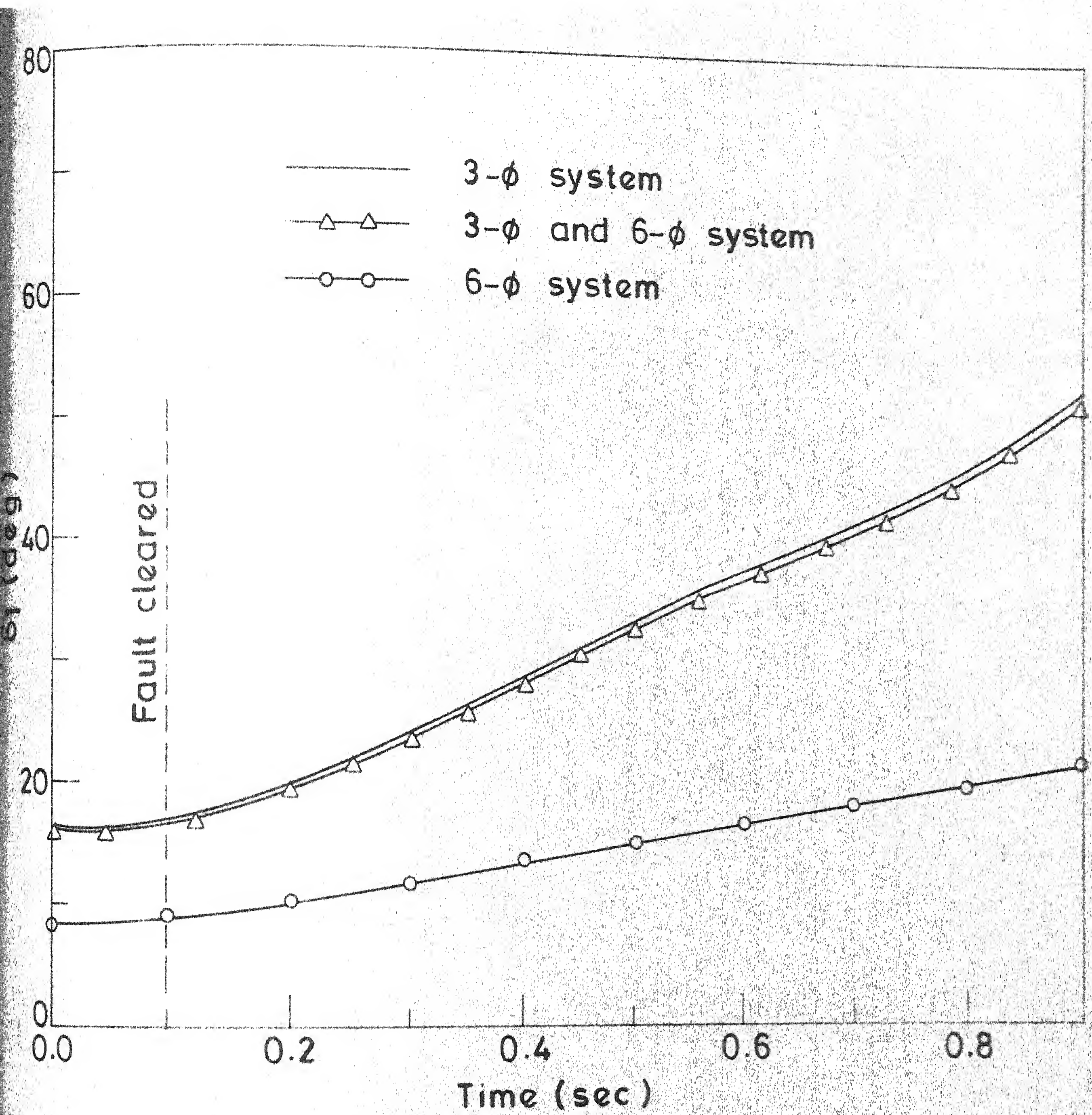


Fig. 5.2(a) Variation of δ_1 with time.

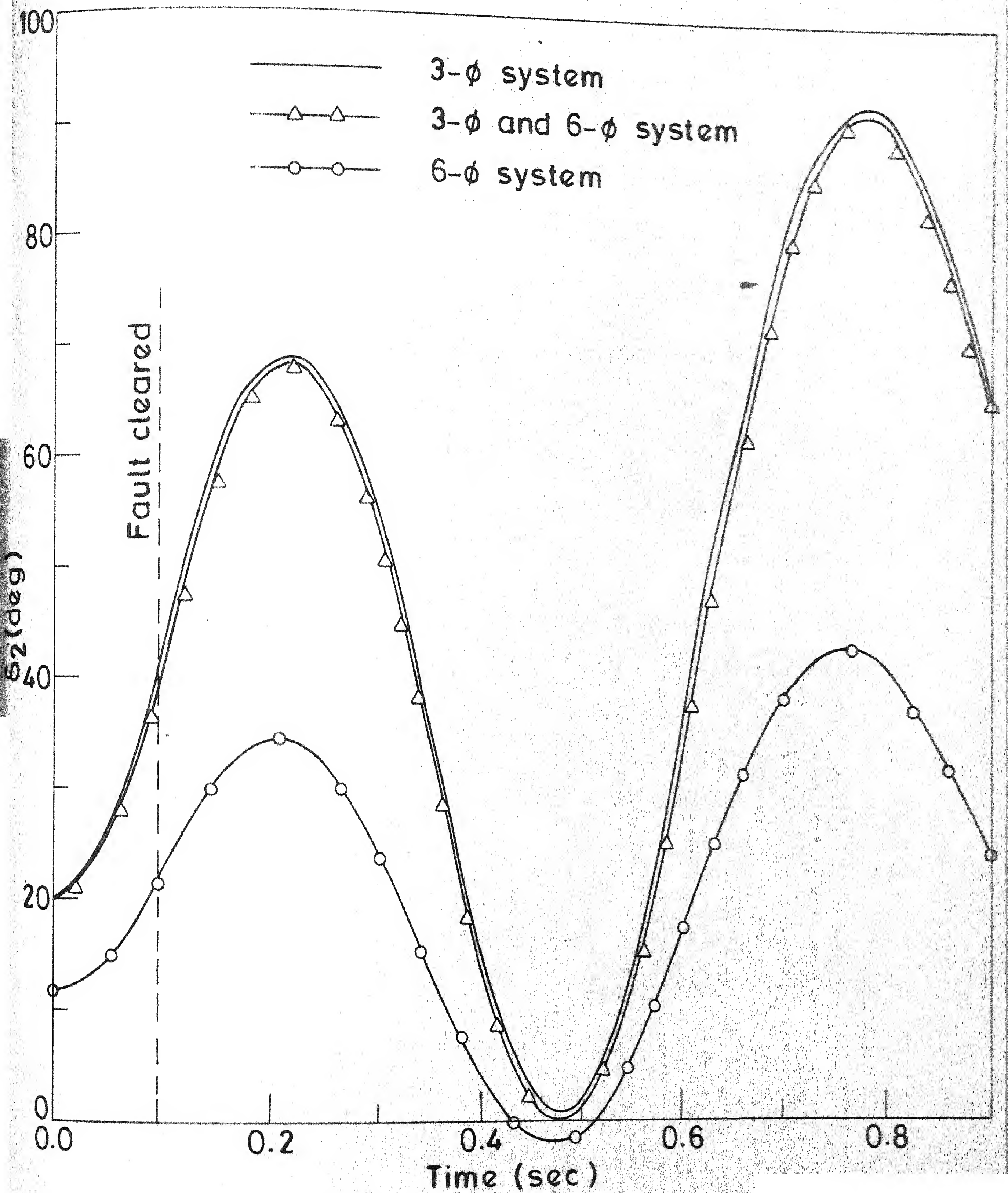


Fig. 5.2 (b) Variation of δ_2 with time.

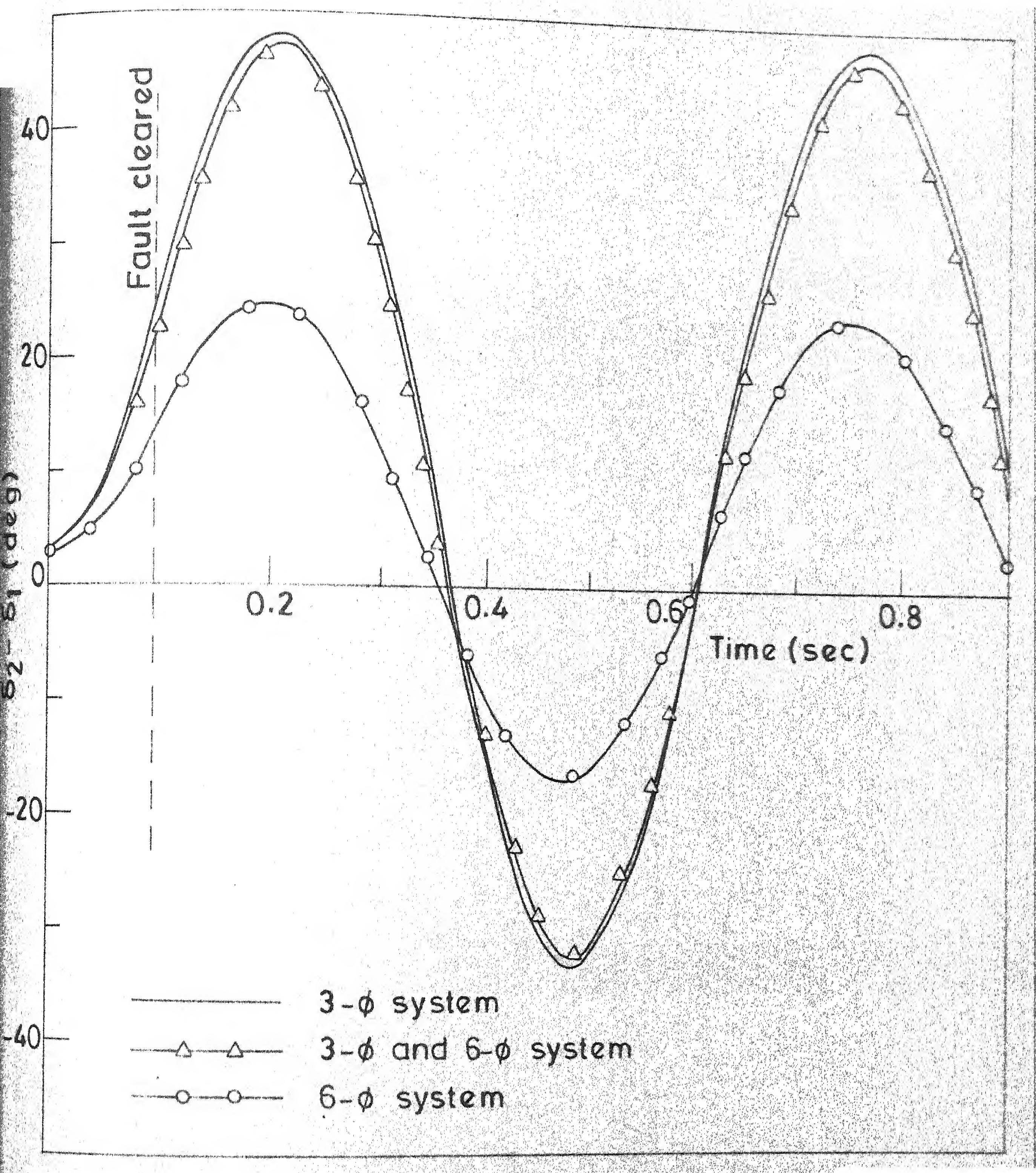


Fig. 5.2 (c) Variation of $\delta_2 - \delta_1$ with time.

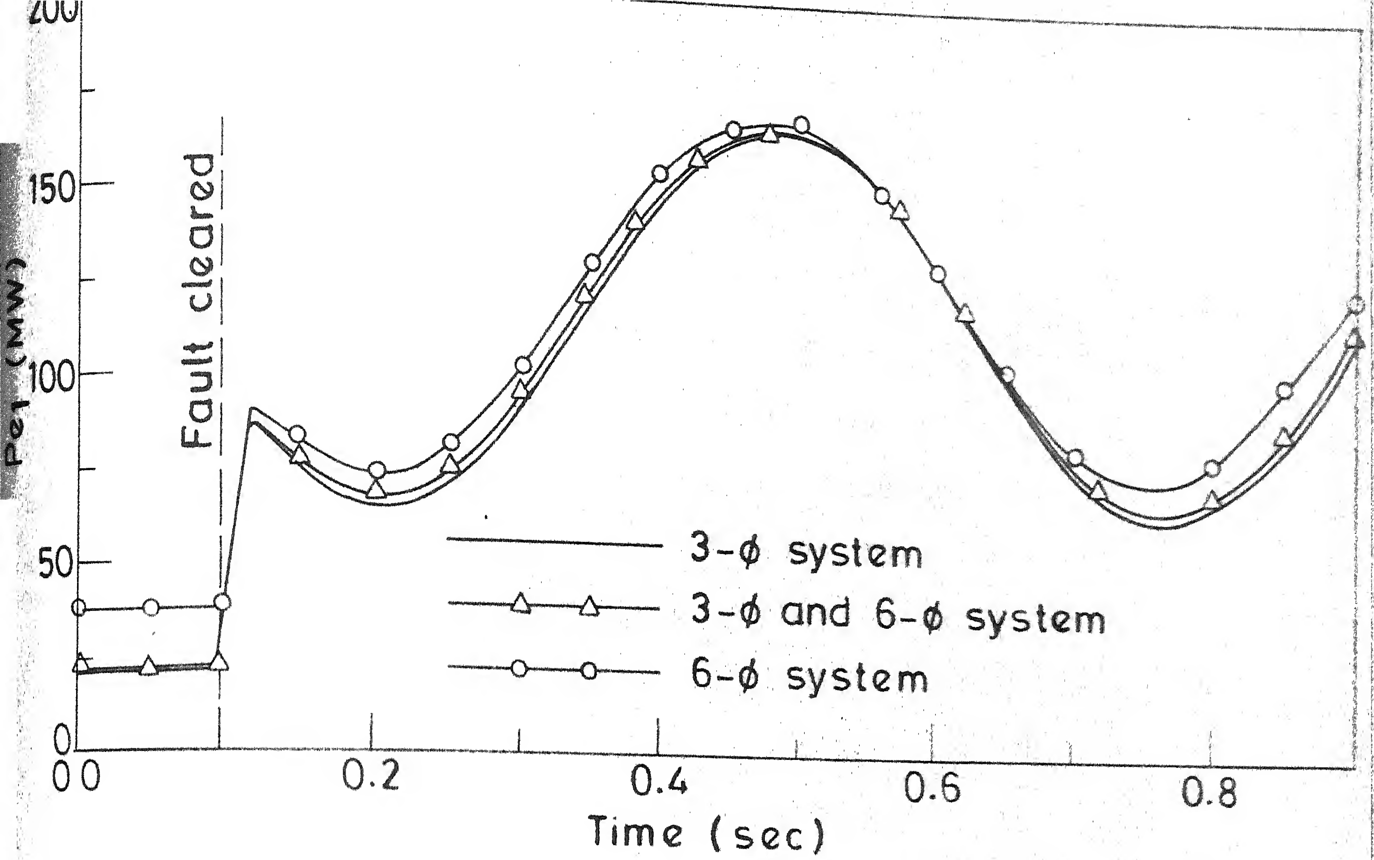


Fig. 5.2 (d) Variation of Pe_1 with time.

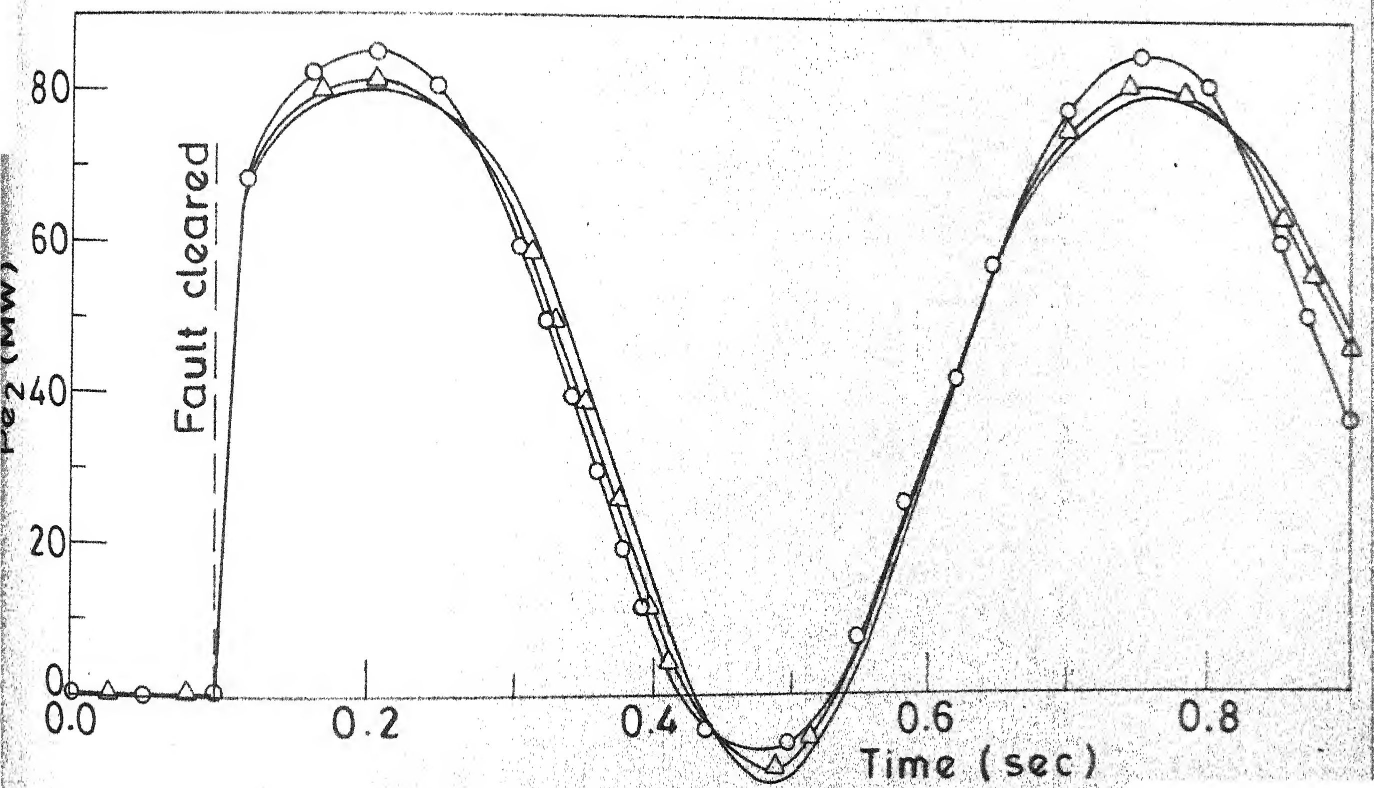


Fig. 5.2 (e) Variation of Pe_2 with time.

II. Transient stability investigations for faults in a six-phase line using phase coordinate method

To illustrate the procedure of phase coordinate method, the transient stability behaviour of the sample system of Fig.5.1 (redrawn in Fig. 5.3 for conceptual clarity) with line 5 as six-phase line is investigated for various faults occurring on six-phase line. However, by way of demonstration, only two cases of faults namely L-G fault at bus 16, and LLLLLL-G fault at buses (16-21) are reported here. The prefault conditions are given in Table 5.5 and the plot of δ_1, δ_2 and $(\delta_2 - \delta_1)$ as functionsof time are depicted in Fig. 5.4 (curves a,b and c). The duration of fault is considered as 0.1 sec. in both the cases.

Table 5.5

Prefault conditions of network of Fig. 5.3

Bus No.	Type of bus	Voltage $\|V\|$ (p.u.)	Voltage Angle (deg)
1	1	1.060	0.000
2	1	1.060	-120.000
3	1	1.060	120.000
4	3	1.050	-2.864
5	3	1.050	-122.839
6	3	1.050	117.255
7	3	1.029	-5.058
8	3	1.029	-125.032
9	3	1.029	115.045

contd...

Bus No.	Type of bus	Voltage V (p.u.)	Voltage Angle (deg)
10	3	1.029	-5.398
11	3	1.029	-125.369
12	3	1.029	114.708
13	3	1.034	-6.318
14	3	1.034	-126.273
15	3	1.034	113.797
16	6	1.048	-4.006
17	6	1.048	-63.888
18	6	1.048	-123.970
19	6	1.048	175.996
20	6	1.048	116.113
21	6	1.048	56.029
22	6	1.038	-5.157
23	6	1.038	-65.040
24	6	1.038	-125.118
25	6	1.038	174.844
26	6	1.038	114.961
27	6	1.038	54.880

where Type of Bus 1,3 and 6 indicate buses corresponding to slack, three-phase and six-phase respectively.

III. Relative performance of six-phase lines

In order to study the relative performance of a six-phase line and also the impact of converting certain existing double circuit three-phase lines to six-phase lines, various faults were applied at bus 2 (corresponding to the most severe disturbance) and also at bus 4 (corresponding to a bus fault where six-phase line does not terminate).Lines 1

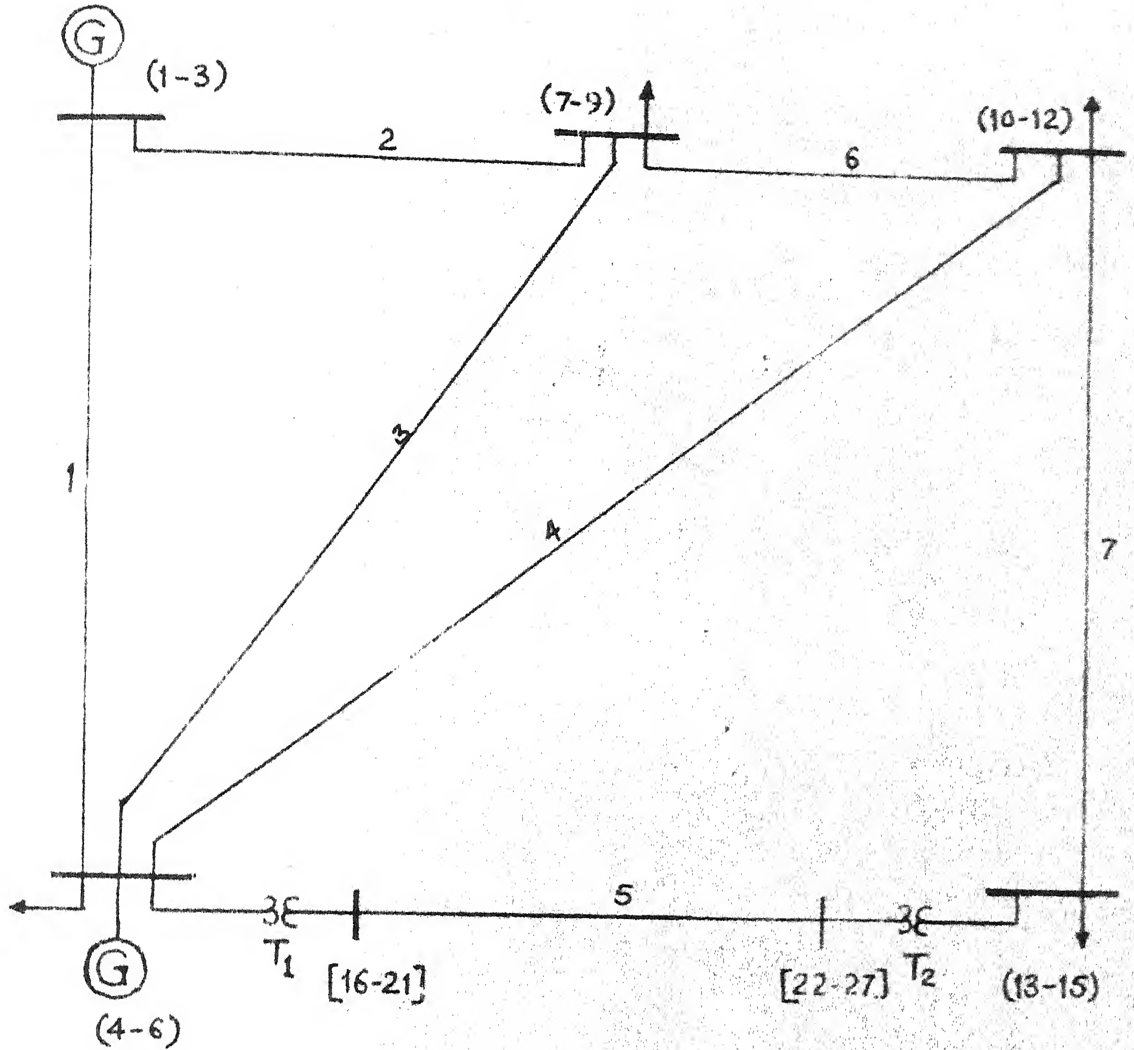


FIG. 5.3 SCHEMATIC REPRESENTATION OF THE SAMPLE SYSTEM OF FIG. 5.1 FOR PHASE COORDINATE METHOD

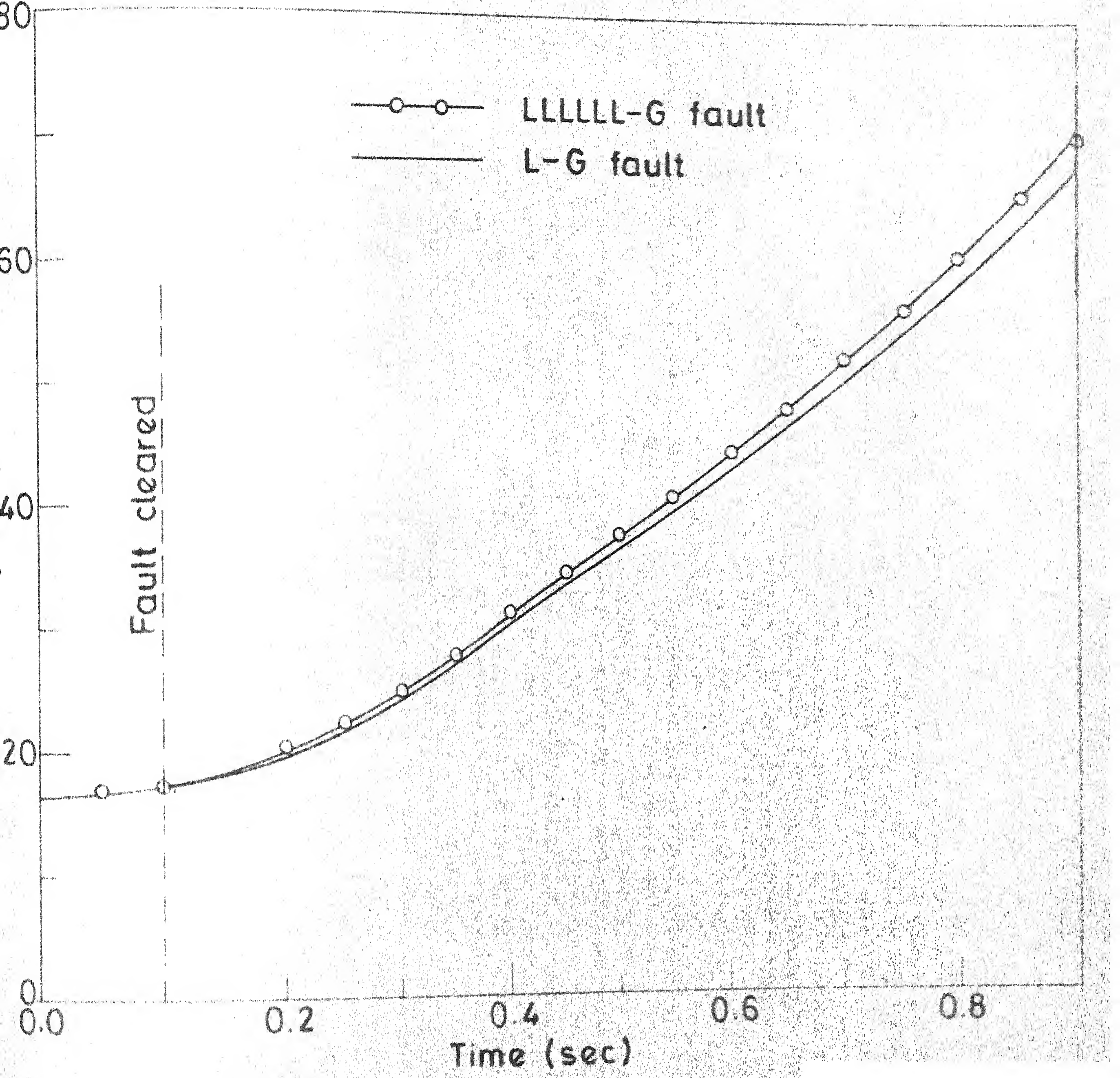
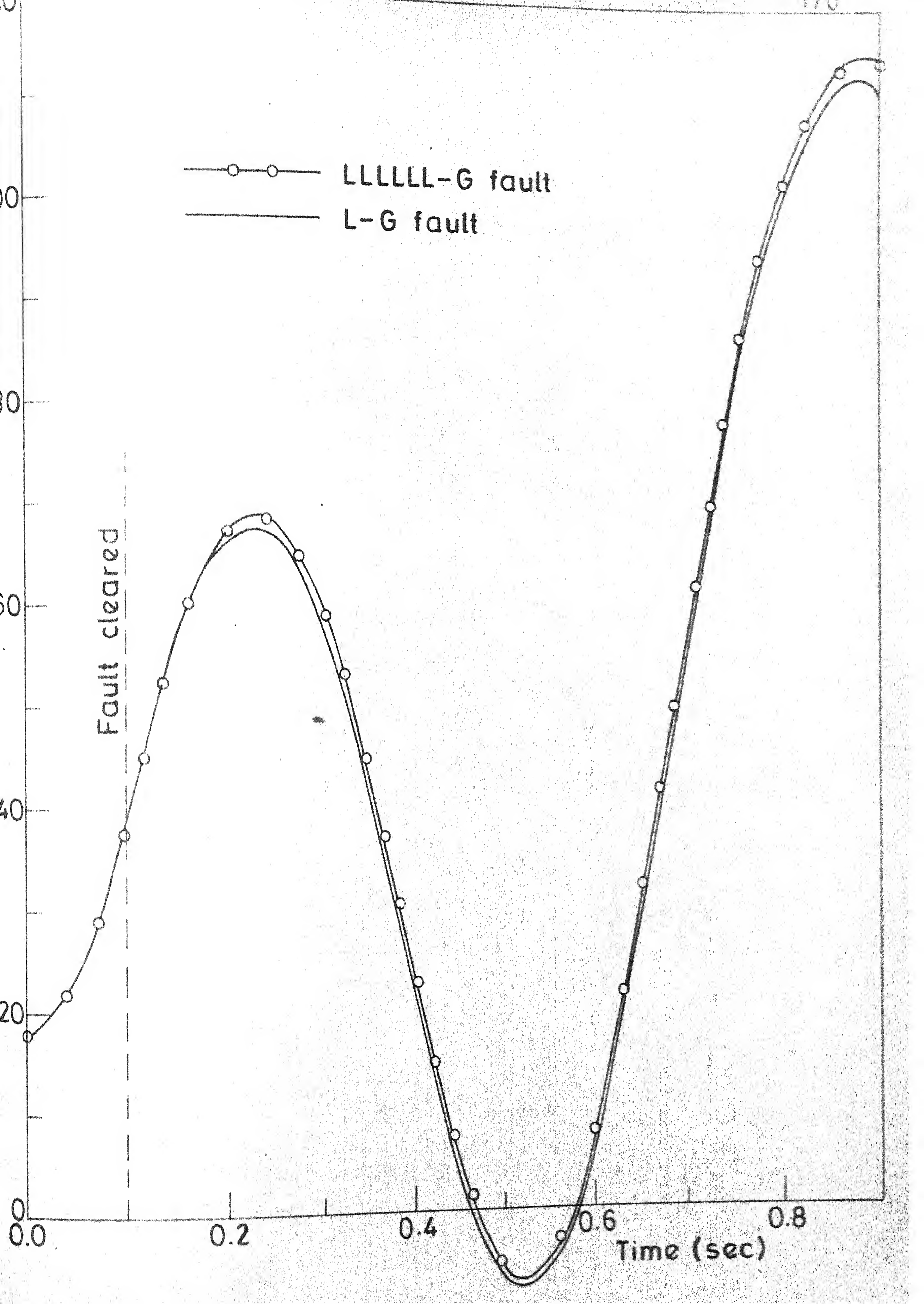


Fig. 5.4(a) Variation of δ_1 with time.



LLLLLL-G fault
L-G fault
Fault cleared
Time (sec)
0.0
0.2
0.4
0.6
0.8
0
20
40
60
80
100

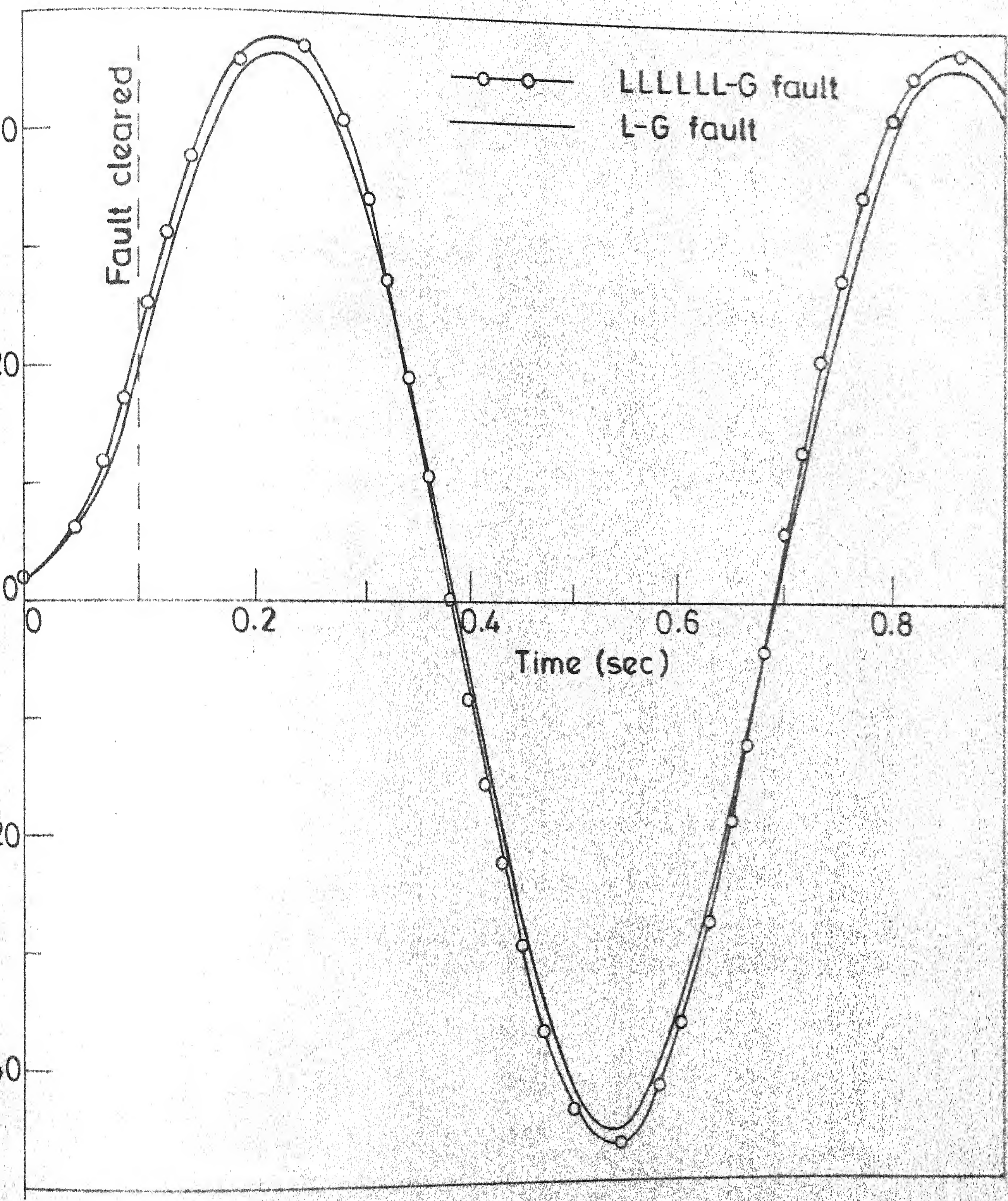


Fig. 5.4(c) Variation of $(\delta_2 - \delta_1)$ with time.

and 5 are considered for conversion to six-phase lines. The critical clearing time as determined for different network conditions during fault and post fault periods are consolidated in Table 5.6.

5.3.3 Discussion of Results

From the results of the various case studies reported in Section 5.3.2, the following observations may be made.

i) The performance of a six-phase system is far better than three-phase system as shown in Fig. 5.2 (curves a,b,c,d and e) for the same loading conditions. Even at higher system loadings, a substantial improvement characterised by lower magnitudes of machine angles and their difference, with six-phase system is noticed. The composite three-phase and six-phase system has a performance comparable to three-phase system as evidenced by the corresponding plots of machine angles and power delivered etc.

ii) A detailed investigation of stability performance under different faults viz. single line to ground (L-G), six-line to ground (LLLLLL-G) etc. on a six-phase line has been demonstrated using the phase coordinate procedure. An examination of the curves for machine angles show that the performance of the system is not significantly different.

Table 5.6

Summary of results for several transient stability studies on sample system (Fig. 5.1)

S.No.	Type of fault	Faulty Bus	Mode of operation of power system	Post fault condition	Critical clearing time (sec.)
1	LLL-G	2	Three phase	Line 5 is tripped out	0.16
2	LLL-G	2	Three phase and six-phase. Line 5 is a 6-φ line	Line 5 is tripped out	0.16
3	LLL-G	2	Three phase	Fault removed	0.16
4	LLL-G	2	Three-phase and six-phase. Line 5 is a 6-φ line	Fault removed	0.16
5	LLL-G	2	Three-phase and six-phase. Line 1 and 5 are 6-φ lines	Fault removed	0.16
6	LLLLLL-G	2	Six-phase . All lines operating as 6-φ lines (configuration C)	Fault removed	0.30
7	LLL-G	4	Three-phase	Fault removed	0.22
8	LLL-G	4	Three-phase and six-phase. Line 5 is a 6-φ line	Fault removed	0.20
9	LLL-G	4	Three-phase and six-phase. Lines 1 and 5 are 6-φ lines	Fault removed	0.20

iii) The relative performance of a six-phase line in terms of critical clearing time is comparable to a double circuit three-phase line as can be seen from the various cases detailed in Table 5.6. With completely six-phase system, the benefit is significantly substantial.

5.4 CONCLUDING REMARKS

In this chapter, three-phase techniques for simulation of transient stability problems have been modified /extended for a six-phase and a composite three-phase and six-phase system. The methods using equivalent single phase as well as phase coordinate representationshave been applied to work out a sample system employing alternate cycles of computation of differential and network performance equations. The modified Euler's method and Gauss-Siedel Iterative techniques have been used for numerical integration and for solving network equations respectively.

A simple and preliminary investigation carried out on a sample system indicatesthat the six-phase system performs better than three-phase system during transient disturbances (faults). Based on this investigation, it is also observed that the performance of six-phase line (converted from a double circuits +hree-phase line) is comparable to a double circuit three-phase line.

CHAPTER VI

CONCLUSIONS

This chapter summarises the work done in this thesis. A few suggestions for further investigation are also included.

6.1 BRIEF REVIEW OF THE WORK DONE

The basic objective of this thesis has been to develop suitable mathematical models and analytical tools for several system studies for six-phase (multi-phase) power systems which are being developed to alleviate certain problems of present day power systems.

Initially, an overview of the feasibility of multi-phase system is discussed based on the studies reported in the literature. An attempt is made to focus the salient features of performance, advances made towards construction, testing and design capabilities, and the areas where multi-phase transmission may find immediate and future applications.

Next, the mathematical modelling of various elements of a six-phase (multi-phase) system suitable for steady-state analysis are presented. Two alternative models for three-phase/six-phase transformers which are required to link six-phase elements with three-phase network are derived. In the first model, the transformers are represented as ideal

transformers in series with their leakage impedances. The second model is obtained from the symmetrical lattice equivalent of a transformer which is conceptualised as a three winding transformer for the purpose. Model I is simple to obtain and reasonably accurate. Model II is more precise than model I and is suitable for rigorous representation of the overall system intended for unbalanced network analysis in phase coordinates. In both the models, the off nominal tappings have been included. The modelling procedures have been applied to several types of three-phase/six-phase and also six-phase transformer viz., wye/star, delta/star, wye/hexagon, delta/hexagon and star/star.

A generalisation of transmission line models to multiphase for short, medium and longlengths in terms of phase impedance/admittance matrix and ABCD-parameters, are carried out. Considering the six-phase line as a network connected between the three-phase buses via interfacing transformers (three-phase/six-phase) at either ends, its three-phase equivalent representation in terms of impedance/admittance matrix and ABCD-parameters are derived. Similarly, the six-phase equivalent representations of a three-phase line are also developed. The symmetrical component transformations of equivalent three-phase and six-phase descriptions are discussed. Equivalent single phase representation of the lines are also derived for balanced system studies. The representation of a six-phase generator (which may become a reality

for high power applications in future) is developed incorporating the unbalances in the machine, particularly, suitable for unbalanced analysis. The six-phase (multi-phase) and also three-phase loads, their equivalent descriptions, and symmetrical component transformations are presented for the purpose of various studies of multi-phase systems.

Employing the representations of various multi-phase elements along with those of usual three-phase elements, a six-phase or a composite three-phase and six-phase system is then modelled : in balanced conditions on an equivalent single-phase basis, and in unbalanced conditions, as an equivalent three-phase/six-phase system and also as a composite three-phase and six-phase system retaining the physical identities of different elements. The procedure is explained with the help of a numerical example.

The various schemes of modelling the overall system in balanced as well as in unbalanced conditions are applied to carry out load flow studies for six-phase and composite multi-phase systems. The procedure of phase coordinate method has been particularly emphasised. Three alternative schemes are evolved to effectively deal with the problem of unbalances either occurring in three-phase or six-phase or both three-phase and six-phase part of the network in a composite multi-phase power system. The procedure are demonstrated with help of numerical examples.

The problem and complexities of faults in six-phase systems and the techniques for their analysis have been discussed. A straightforward extension of symmetrical component transformation technique for three-phase system employing bus impedance description of the network is carried out for six-phase system. The approach eliminates the several limitations of the existing techniques. The phase coordinate method which is most suited for unbalanced network and simultaneous faults is delineated for a six-phase and also for a composite multi-phase network. The various procedures developed to this end, are illustrated with the help of numerical examples.

The transient stability investigations for a six-phase and also a composite three-phase and six-phase systems, are carried out by modifying/extending three-phase techniques. The methods using equivalent single phase as well as phase coordinate representations are applied to obtain the solutions employing alternate cycles of computation of differential and network performance equations.

Based on the various case studies carried out on six-phase and composite three-phase and six-phase systems, the following observations regarding the performance of high-phase order systems are made.

i) Load Flow

The high-phase order systems indicate their potentiality

to maintain better voltage magnitudes and phase angles even at higher system loadings than those of conventional three-phase system. When an existing double circuit three-phase line is converted to a six-phase line (with same line to line voltage and conductor configurations), the voltage regulation and line efficiency are improved. In addition, the capability of transmission may be increased to 1.732 times. When number of such lines are uprated to six-phase lines, the benefits in terms of voltage magnitudes and phase angle and line efficiencies increase. The real and reactive line loadings for the same power transmitted are considerably reduced with six-phase operation of lines. However, the distribution of these benefits is governed by the network topology and locations of generating and load centers. The most important impact of change over from three-phase to six-phase operation of double circuit line, is that, higher system loads can be met without major reconstruction of the lines.

ii) Faults

The performance of six-phase system during faults is similar to a three-phase system. However, a composite three-phase and six-phase system shows different performance during ground short circuits at six-phase buses from other system configurations. With particular reference to a case where six-phase line is integrated to the three-phase system via wye/star transformers at both ends, it is found that when only

one phase conductor of the line, out of the two phases having 180^{o} phase displacement, is involved, the ground fault current magnitude is nearly double than that when both phases are involved. This situation does not take place when the fault occurs in the three-phase part of the network far away from three-phase/six-phase transformers.

iii) Transient Stability

The transient stability performance of six-phase system has been found to be better than that of three-phase systems. However, the performance of a composite three-phase and six-phase system (with few lines converted from double circuit three-phase to six-phase line) is comparable to three-phase system.

6.2 SCOPE FOR FURTHER WORK

i) The modelling of three-phase/six-phase transformers is one of the very important aspects in the analysis of a composite three-phase and six-phase system. It would be worthwhile to extend the procedure to other types of transformer connections schemes with such details as, effect of core and saturation, neutral connections and the phase shifting, etc.

ii) The load flow analysis has been carried out employing Gauss-Siedel iterative method in the present investigation. It is desirable to develop the procedures with stronger convergence characteristics. In addition, the load flow performance of six-phase (multi-phase) lines may be examined with various conductor configurations and tower geometries to obtain the most appropriate and practical designs for six-phase lines. The cases involving mutual couplings, untransposed lines and earthing arrangements of different equipments of the system may also be included for the purpose of study.

iii) The fault investigations are carried out by assuming the transmission lines fully transposed in this case. As pointed out, the six-phase lines are difficult to transpose in practice, hence the fault analyses need be carried out using realistic data drawn from field tests and with other transformer (three-phase/six-phase) connection schemes.

iv) A very simple approach for transient stability investigations of composite multi-phase system has been taken in the present case. It is desirable to include other machine details such as field flux decay, saturation effects etc. and possibly with more accurate/faster solution techniques.

v) Although the construction of experimental multi-phase lines, their testing and network simulator studies are reported to be carried out by P.T.I. (Power Technologies Inc.) and Allegheny Power Company, USA, there is a need to go in for more such experimentation in order to gain experience in operation and recognise newer problems.

APPENDIX A

SIX-PHASE MACHINE MODEL

The general network element shown in Fig.A1 is described by the nodal eqn.(A1),

$$V_p^6 - V_p'^6 = E_p^6 + Z_p^6 I_p'^6 \qquad (A.1)$$

where

$$V_p'^6 = [V_7 \ V_8 \ V_9 \ V_{10} \ V_{11} \ V_{12}]^T$$

$$V_p^6 = [V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ V_6]^T ; E_p^6 = [E_a \ E_b \ E_c \ E_d \ E_e \ E_f]^T$$

$$I_p'^6 = [I_a \ I_b \ I_c \ I_d \ I_e \ I_f]^T$$

The nodal equations for the current and voltage directions shown, can be written from (A.1) as

$$\begin{bmatrix} I_p^6 \\ I_p'^6 \end{bmatrix} = \begin{bmatrix} Y_p^6 & -Y_p^6 & -Y_p^6 \\ -Y_p^6 & Y_p^6 & Y_p^6 \end{bmatrix} \begin{bmatrix} V_p^6 \\ V_p'^6 \\ E_p^6 \end{bmatrix} \qquad (A.2)$$

Consider the internal emf vector of the machine E_p^6 to be balance such that

$$E_p^6 = [1 \quad \alpha^* \quad -\alpha \quad -1 \quad -\alpha^* \quad \alpha] \ E_a$$

The Nodes 7-12 are joined to form a neutral N such that,

$$I_N = I_7 + I_8 + I_9 + I_{10} + I_{11} + I_{12} \qquad (A.3)$$

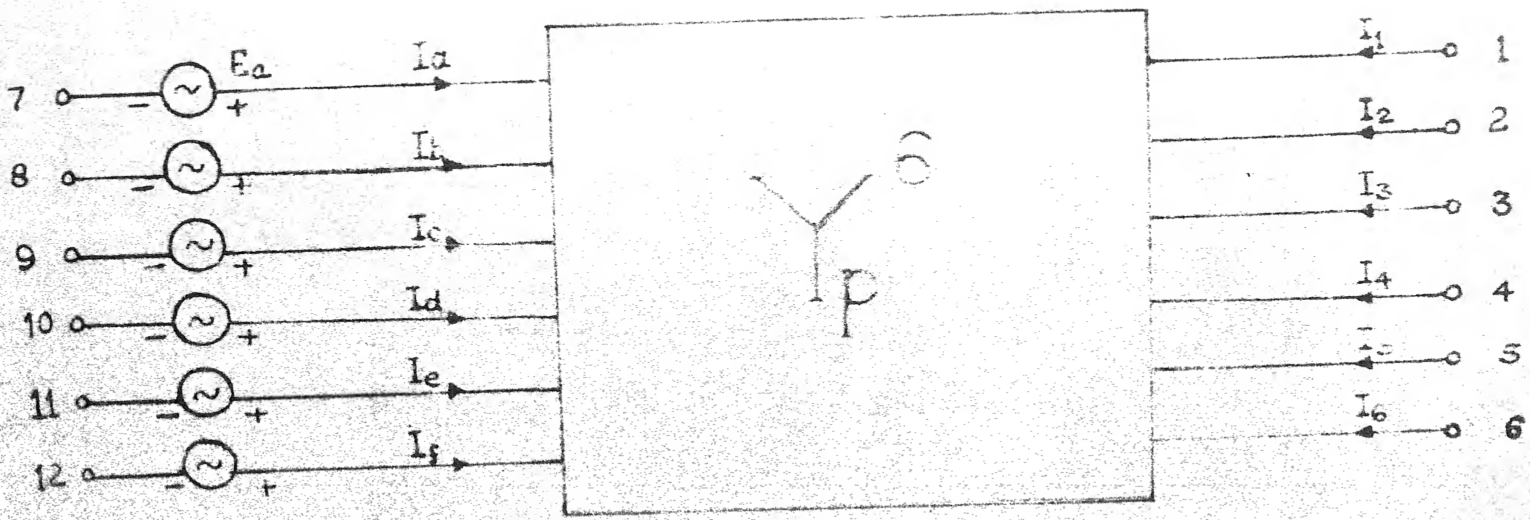


FIG. A.1 A GENERAL SIX-PHASE NETWORK ELEMENT

and

$$V_N = V_7 = V_8 = \ldots = V_{12} \tag{A.4}$$

Then the internal voltage vector becomes,

$$E_p^6 = [E_a+V_N,\ E_b+V_N,\ \ldots\ E_f+V_N]^T \tag{A.5}$$

and the current vector $I_p'^6$ given by

$$I_a = \frac{S_a^*}{(V_N+E_a)^*} \; ; \quad I_b = \frac{S_b^*}{(V_N+E_b)^*} ; \ \ldots \quad I_f = \frac{S_f^*}{(V_N+E_f)^*} \tag{A.6}$$

where S_a, S_b, ..., S_f are powers corresponding to phases a,b,c, ..., f respectively.

Before making the substitution of the above information in eqn. (A.1), it may be noted that

$$Y_p^6 = \begin{bmatrix} Y_S & Y_{m1} & Y_{m2} & Y_{m3} & Y_{m4} & Y_{m5} \\ Y_{m5} & Y_S & Y_{m1} & Y_{m2} & Y_{m3} & Y_{m4} \\ Y_{m4} & Y_{m5} & Y_S & Y_{m1} & Y_{m2} & Y_{m3} \\ Y_{m3} & Y_{m4} & Y_{m5} & Y_S & Y_{m1} & Y_{m2} \\ Y_{m2} & Y_{m3} & Y_{m4} & Y_{m5} & Y_S & Y_{m1} \\ Y_{m1} & Y_{m2} & Y_{m3} & Y_{m4} & Y_{m5} & Y_S \end{bmatrix} \tag{A.7}$$

where

$$Y_S = \frac{1}{6}\,(y_o + y_1 + y_2 + y_3 + y_4 + y_5)$$

$$Y_{m1} = \frac{1}{6}\,(y_o + \alpha y_1 - \alpha^* y_2 - y_3 - \alpha y_4 + \alpha^* y_5)$$

$$Y_{m2} = \frac{1}{6}\,(y_o - \alpha^* y_1 - \alpha y_2 + y_3 - \alpha^* y_4 - \alpha y_5)$$

$$Y_{m3} = \frac{1}{6}\,(y_o - y_1 + y_2 - y_3 + y_4 - y_5)$$

$$Y_{m4} = \frac{1}{6}\,(y_o - \alpha y_1 - \alpha^* y_2 + y_3 - \alpha y_4 - \alpha^* y_5)$$

$$Y_{m5} = \frac{1}{6}\,(y_o + \alpha^* y_1 - \alpha y_2 - y_3 - \alpha^* y_4 + \alpha y_5)$$

Substituting E_p^6 and $I_p'^6$ vide eqns. (A.5) and (A.6) in (A.1) and upon collection of terms, the following equations results

$$\left[\begin{array}{ccc} & -y_o & -y_1 \\ & -y_o & -\alpha^* y_1 \\ Y_p^6 & -y_o & \alpha y_1 \\ & -y_o & y_1 \\ & -y_o & \alpha^* y_1 \\ & -y_o & -\alpha y_1 \\ \hline & y_o & y_1 \\ & y_o & \alpha^* y_1 \\ -Y_p^6 & y_o & -\alpha y_1 \\ & y_o & -y_1 \\ & y_o & -\alpha^* y_1 \\ & y_o & \alpha y_1 \end{array}\right] \left[\begin{array}{c} \\ V_p^6 \\ \\ \hline V_N \\ \hline E_a \\ \\ \end{array}\right] = \left[\begin{array}{c} I_p^6 \\ \hline S_a^*/(V_N+E_a)^* \\ S_b^*/(V_N+E_b)^* \\ S_c^*/(V_N+E_c)^* \\ S_d^*/(V_N+E_d)^* \\ S_e^*/(V_N+E_e)^* \\ S_f^*/(V_N+E_f)^* \\ \\ \end{array}\right] \qquad (A.8)$$

In addition, for node N, an equation may be written utilizing (A.2) and (A.1) as

$$[-y_o \quad -y_o \quad -y_o \quad -y_o \quad -y_o \quad -y_o \quad 6y_o \quad 0] \begin{bmatrix} V_p^6 \\ V_N \\ E_a \end{bmatrix} = I_N \quad \text{(A.9)}$$

Cross multiplication of eqn. (A.8) and noting that

$$E_b^* = \alpha E_a^*, \quad E_c^* = -\alpha^* E_a^*, \quad E_d^* = -E_a^*, \quad E_e^* = -\alpha E_a^*, \quad E_f^* = \alpha^* E_a^*$$

$$\frac{1}{6}[-Y_S\, V_1\, (V_N^*+E_a^*)-Y_{m1}\, V_2(V_N^*+E_a^*)-Y_{m2}V_3(V_N^*+E_a^*)-Y_{m3}V_4(V_N^*+E_a^*) - $$
$$-Y_{m4}\, V_5(V_N^*+E_a^*)-Y_{m5}\, V_6(V_N^*+E_a^*)]+y_o\, V_N(V_N^*+E_a^*)+y_1E_a(V_N^*+E_a^*) = S_a^* \quad \text{(A.10)}$$

$$\frac{1}{6}[-Y_{m5}\, V_1(V_N^*+\alpha E_a^*)-Y_SV_2(V_N^*+\alpha E_a^*)-Y_{m1}V_3(V_N^*+\alpha E_a^*)-Y_{m2}V_4(V_N^*+\alpha E_a^*) - $$
$$-Y_{m3}V_5(V_N^*+\alpha E_a^*)- Y_{m4}V_6(V_N^*+\alpha E_a^*)]+y_oV_N(V_N^*+\alpha E_a^*)+\alpha\, y_1E_a(V_N^*+\alpha E_a^*)=S_b^* \quad \text{(A.11)}$$

$$\frac{1}{6}[-Y_{m4}V_1(V_N^*-\alpha^*E_a^*)-Y_{m5}V_2(V_N^*-\alpha^*E_a^*)-Y_SV_3(V_N^*-\alpha^*E_a^*)-Y_{m1}V_4(V_N^*-\alpha^*E_a^*) - $$
$$-Y_{m2}V_5(V_N^*-\alpha^*E_a^*)-Y_{m3}V_6(V_N^*-\alpha^*E_a^*)]+y_oV_N(V_N^*-\alpha^*E_a^*)-\alpha y_1E_a(V_N^*-\alpha^*E_a^*)$$
$$= S_c^* \quad \text{(A.12)}$$

$$\frac{1}{6}\, [-Y_{m3}V_1(V_N^*-E_a^*)-Y_{m4}V_2(V_N^*-E_a^*)-Y_{m5}V_3(V_N^*-E_a^*)-Y_SV_4(V_N^*-E_a^*) - $$
$$-Y_{m1}V_5(V_N^*-E_a^*)-Y_{m2}V_6(V_N^*-E_a^*)]+y_oV_N(V_N^*-E_a^*)-y_1E_a(V_N^*-E_a^*) = S_d^* \quad \text{(A.13)}$$

$$\frac{1}{6}\,[-Y_{m2}V_1(V_N^*-\alpha E_a^*)-Y_{m3}V_2(V_N^*-\alpha E_a^*)-Y_{m4}V_3(V_N^*-\alpha E_a^*)-Y_{m5}V_4(V_N^*-\alpha E_a^*)-$$
$$-Y_SV_5(V_N^*-\alpha E_a^*)-Y_{m1}V_6(V_N^*-\alpha E_a^*)]+y_oV_N(V_N^*-\alpha E_a^*)-\alpha^*y_1E_a(V_N^*-\alpha E_a^*)=S_e^* \qquad (A.14)$$

$$\frac{1}{6}\,[-Y_{m1}V_1(V_N^*+\alpha^*E_a^*)-Y_{m2}V_2(V_N^*+\alpha^*E_a^*)-Y_{m3}V_3(V_N^*+\alpha^*E_a^*)-Y_{m4}V_4(V_N^*+\alpha^*E_a^*)$$
$$-Y_{m5}V_5(V_N^*+\alpha^*E_a^*)]+y_oV_N(V_N^*+\alpha^*E_a^*)+\alpha y_1E_a(V_N^*+\alpha^*E_a^*) = S_f^* \qquad (A.15)$$

Adding eqns. (A.10) - (A.15) together and substituting the values of $Y_S, Y_{m1}, Y_{m2}, Y_{m3}, Y_{m4}$ and Y_{m5} gives

$$-y_oV_1V_N^*-y_oV_2V_N^*-y_oV_3V_N^*-y_oV_4V_N^*-y_oV_5V_N^*-y_oV_6V_N^*+6y_oV_NV_N^* - y_1V_1E_a^*$$
$$-\alpha y_1V_2E_a^* + \alpha^*y_1V_3E_a^* + y_1V_4E_a^* + \alpha y_1V_5E_a^* - \alpha^*y_1V_6E_a^* +$$
$$+6y_1E_a^*E_a^* = S_a^*+S_b^*+S_c^*+S_d^*+S_e^*+S_f^* \qquad (A.16)$$

Examination of (A.16) reveals that the first 7 terms of (A.16) are eqn. (A.9) V_N. With the usual condition where no current is injected into the neutral node, i.e. $I_N = 0$, eqn. (A.16) becomes

$$[-y_1 \;\; -\alpha y_1 \;\; \alpha^*y_1 \;\; y_1 \;\; \alpha y_1 \;\; -\alpha^*y_1 \;\; 6y_1]\begin{bmatrix} V_p^6 \\ E_a \end{bmatrix} = \frac{S_a^*+S_b^*+S_c^*+S_d^*+S_e^*+S_f^*}{E_a^*} \qquad (A.17)$$

For the general case, however, where the generator neutral is earthed via an impedance Y_N eqn. (A.2) is modified to

$$I_N = I_7+I_8+I_9+I_{10}+I_{11}+I_{12}+y_NV_N$$

$$= -y_oV_1-y_oV_2-y_oV_3-y_oV_4-y_oV_5-y_oV_6+(6y_o+y_N)V_N \qquad (A.18)$$

and hence

$$V_N^*I_N = -y_oV_1V_N^*-y_oV_2V_N^*-y_oV_3V_N^*-y_oV_4V_N^*-y_oV_5V_N^*-y_oV_6V_N^*+(6y_o+y_N)V_NV_N^* \qquad (A.19)$$

Substituting $V_N^*I_N$ in (A.17), again with $I_N = 0$, gives

$$-y_NV_NV_N^*+[-y_1 \quad -\alpha y_1 \quad \alpha^* y_1 \quad y_1 \quad \alpha y_1 \quad -\alpha^* y_1 \quad 6y_1]\begin{bmatrix} V_p^6 \\ E_a \end{bmatrix} E_a^* = S_a^*+S_b^*+S_c^*+S_d^*+S_e^*+S_f^* \qquad (A.20)$$

The general equation relating total power to voltages is then

$$[-y_1 \quad -\alpha y_1 \quad \alpha^*y_1 \quad y_1 \quad \alpha y_1 \quad -\alpha^*y_1 \quad 6y_1]\begin{bmatrix} V_p^6 \\ E_a \end{bmatrix} = \frac{S_a^*+S_b^*+S_c^*+S_d^*+S_e^*+S_f^*}{E_a^*} + \frac{y_N \; |V_N|^2}{E_a^*} \qquad (A.21)$$

The machine equations collected together are, therefore, with $I_N = 0$, and combining the first six equations of (A.8), (A.9) and (A.21), thus,

$$\left[\begin{array}{cccccc|c|c} & & & & & & -y_1 & -y_o \\ & & & & & & -\alpha^* y_1 & -y_o \\ & & Y_p^6 & & & & \alpha y_1 & -y_o \\ & & & & & & y_1 & -y_o \\ & & & & & & \alpha\ y_1 & -y_o \\ & & & & & & -\alpha y_1 & -y_o \\ \hline -y_1 & -\alpha y_1 & \alpha^* y_1 & y_1 & \alpha y_1 & -\alpha^* y_1 & 6y_1 & 0 \\ \hline -y_o & -y_o & -y_o & -y_o & -y_o & -y_o & 0 & 6y_o+Y_N \end{array}\right] \left[\begin{array}{c} \\ \\ V_p^6 \\ \\ \\ \hline E_a \\ \hline V_N \end{array}\right] = \left[\begin{array}{c} \\ \\ I_p^6 \\ \\ \\ \hline (S^*+y_N|V_N|^2)/E_a^* \\ \hline 0 \end{array}\right]$$

(A.22)

The terms $\dfrac{y_N|V_N|^2}{E_a^*}$ being smaller than S^*/E_a^*, to a first degree approximation, can be neglected. Thus eqn. (A.22) can be written as,

$$\begin{bmatrix} Y_p^6 & -\overline{Y}_1 & -\overline{Y}_o \\ -\overline{Y}_2 & 6y_1 & 0 \\ -\overline{Y}_o^T & 0 & (Y_N+6y_o) \end{bmatrix} \begin{bmatrix} V_p^6 \\ E_a \\ V_N \end{bmatrix} = \begin{bmatrix} I_p^6 \\ S^*/E_a^* \\ 0 \end{bmatrix} \qquad \text{(A.23)}$$

where

$$\overline{Y}_1 = [1 \quad \alpha^* \quad -\alpha \quad -1 \quad -\alpha^* \quad \alpha]^T\ y_1$$

$$\overline{Y}_2 = [1 \quad \alpha \quad -\alpha^* \quad -1 \quad -\alpha \quad \alpha^*]y_1$$

$$\overline{Y}_o = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]^T\ y_o$$

APPENDIX B

EQUIVALENT SINGLE-PHASE (REPRESENTATION) CALCULATIONS FOR A COMPOSITE THREE-PHASE AND SIX-PHASE SYSTEM

B.1 EQUIVALENT SINGLE PHASE REPRESENTATION (DERIVED FROM THREE-PHASE EQUIVALENT REPRESENTATION)

The six-phase line L1 (Fig. 3.1) is first converted into an equivalent three-phase line using eqn. (2.26). The equivalent admittance matrix of the line is obtained as,

$$Y^3_{p,eq} = Y^3_{L1} = j \begin{bmatrix} -3.959 & 0 & 0 \\ 0 & -3.959 & 0 \\ 0 & 0 & -3.959 \end{bmatrix} \quad (B.1)$$

In obtaining matrix (B.1), the phase admittance parameter matrix of six-phase line calculated from the knowledge of sequence admittances and the use of relation (2.2) as given by (B.2) is employed.

$$Y^6_p = j \begin{bmatrix} -10.787 & 1.713 & 1.713 & 1.713 & 1.713 & 1.713 \\ 1.713 & -10.787 & 1.713 & 1.713 & 1.713 & 1.713 \\ 1.713 & 1.713 & -10.787 & 1.713 & 1.713 & 1.713 \\ 1.713 & 1.713 & 1.713 & -10.787 & 1.713 & 1.713 \\ 1.713 & 1.713 & 1.713 & 1.713 & -10.787 & 1.713 \\ 1.713 & 1.713 & 1.713 & 1.713 & 1.713 & -10.787 \end{bmatrix}$$

The line L1 is represented on equivalent single phase basis by positive sequence admittance (-j3.959) as obtained from (B.1). In addition, employing the positive sequence admittances of lines L2 and L3, the composite three-phase and six-phase system of Fig. 3.1 is described by the nodal admittance matrix as

$$Y = j\begin{bmatrix} -16.071 & 3.571 & 12.5 \\ 3.571 & -25.000 & 12.5 \\ 12.5000 & 12.500 & -25.0 \end{bmatrix} \qquad (B.3)$$

An alternative representation derived from the equivalent six-phase representation of the entire system follows in Section B.2.

B.2 EQUIVALENT SINGLE-PHASE REPRESENTATION (DERIVED FROM EQUIVALENT SIX-PHASE REPRESENTATION)

For deriving the equivalent single-phase representation of the system in this case, lines L2 and L3 are first converted into six-phase lines. Using eqns. (2.54) with the assumption of ideal six-phase/three-phase transformers, the phase admittance matrix $Y^{s}_{p,eq}$ is given by,

$$Y^{6}_{p,eq}=\frac{j}{2}\begin{bmatrix} -9.666 & -2.833 & 2.833 & 9.666 & 2.833 & -2.833 \\ -2.833 & -9.666 & -2.833 & 2.833 & 9.666 & 2.833 \\ 2.833 & -2.833 & -9.666 & -2.833 & 2.833 & 9.666 \\ 9.666 & 2.833 & -2.833 & -9.666 & -2.833 & 2.833 \\ 2.833 & 9.666 & 2.833 & -2.833 & -9.666 & -2.833 \\ -2.833 & 2.833 & 9.666 & 2.833 & -2.833 & -9.666 \end{bmatrix} \tag{B.4}$$

In obtaining (B.4), the phase admittance matrices Y^3_p for the two identical lines L2 and L3 (derived from the transformation of sequence components to phase quantities employing equations similar to(2.2) were used as shown.

$$Y^3_p= Y^3_{L2}=Y^3_{L3}=j\begin{bmatrix} -9.666 & 2.833 & 2.833 \\ 2.833 & -9.666 & 2.833 \\ 2.833 & 2.833 & -9.666 \end{bmatrix} \tag{B.5}$$

The positive sequence admittances as obtained from (B.4) for lines L2 and L3 equal -j12.5. Employing the positive sequence admittance of six-phase line L1 (including the terminal transformers T_1 and T_2) as -j3.571 together with those of equivalent six-phase lines L2 and L3, the nodal admittance matrix of the overall network is obtained as,

$$Y = j\begin{bmatrix} -16.071 & 3.571 & 12.5 \\ 3.571 & -25.000 & 12.5 \\ 12.500 & 12.500 & -25.0 \end{bmatrix} \qquad (B.6)$$

It is to noted that the representation of the overall system [(B.3) and (B.6)] in both alternative ways is identical as expected since the parameters of the lines given in data refer to the same base.

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